First Edition
Exceedingly Rare

THE THEORY OF PERSPECTIVE DEMONSTRATED; IN A METHOD ENTIRELY NEW.

By which the ſeveral

PLANES, LINES, AND POINTS,

USED IN THIS ART,

ARE SHEWN BY

MOVEABLE SCHEMES,

In the true Poſitions in which they are to be conſidered.

Invented, and now publiſhed for the Uſe of

The ROYAL ACADEMY at WOOLWICH.

BY JOHN LODGE COWLEY,

PROFESSOR OF MATHEMATICKS.

LONDON,

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TO THE MOST HONOURABLE

JOHN MANNERS,

MARQUIS OF GRANBY,

One of His Majeſty's Moſt Honourable Privy Council,
Lord Lieutenant Cuſtos Rotulorum of the County of Derby,
Colonel of the Royal Regiment of Horſe Guards,
Lieutenant General of His Majeſty's Forces, and
Maſter General of the Ordnance, &c. &c. &c.

MY LORD,

THE great Honour Your Lordſhip has condeſcended to beſtow upon me in Your acceptance of my endeavours to facilitate the theory or true idea of PERSPECTIVE, warms me to the moſt faithful diſcharge of THIS, and every other duty of my ſtation; well knowing it to be my beſt recommendation to Your Lordſhip's future favour and protection.

I am, with perfect eſteem,

MY LORD,

Your Lordſhip's moſt obedient,

Royal Academy, Woolwich,
May 1, 1765.

And moſt devoted humble ſervant,

John Lodge Cowley.

To Edgar Peacock from
Frank Brangwyn. Christmas 1947

PREFACE.

I Shall not here point out the uſefulneſs of PERSPECTIVE, or the ſuperior excellence of the principles I have endeavoured to explain, ſince the numerous writings of very eminent perſons, employed on this ſubject, are ſufficient teſtimonies of the former; and it is hoped the enſuing Treatiſe will fully evidence the latter. On the other hand, I frankly own, that my performance allows me not any reaſonable claim or pretence to an enlargement of ſcience, by extending its dominions, though this is confeſſedly the chief merit of an author. Inſtructing youth in certain branches of mathematical knowledge, planned and improved by others, is the province to which I am devoted; and to convey the ſenſe of my Lectures to ſtudents, in the moſt eaſy and perſpicuous manner, is the main point I aſpire to.

The quick ſale of the whole impreſſion of my APPENDIX TO EUCLID, and the repeated demands for that treatiſe, are indubitable marks of publick approbation; and have determined me to expedite a ſecond edition of

it, or rather a new and more comprehenſive work, which I believe will prove a conſiderable help to ſuch as engage in that part of mathematical learning; no leſs perſuaded that this my preſent attempt, to explain and facilitate the doctrine of Planes as applicable to Perſpective, will have a ſimilar effect, eſpecially as it has been approved of by THE HONOURABLE SIR CHARLES FREDERICK, SURVEYOR GENERAL OF HIS MAJESTY'S ORDNANCE, who was pleaſed to conſider it in manuſcript; on which foundation I, with the more confidence, ſubmit it to the publick.

J. L. C.

A SUCCINCT

HISTORY OF PERSPECTIVE.

PERSPECTIVE, according to the accounts of hiſtorians, is derived from painting, particularly as employed in theatrical decorations, but was very little known until about the beginning of the ſixteenth century. The deſire of repreſenting on a plane, or flat ſurface, ſuch figures as ſhould produce, on the eyes of the ſpectators, effects ſimilar to thoſe which would be occaſioned by their being ſeen in relief, and at different diſtances, engaged the artiſts of thoſe times to conſider the apparent diminution of magnitude and alterations of poſition, which ſuch different objects exhibit to the eye, according as they are nearer to or farther from the ſame.

In this reſearch it is, at leaſt, very probable, that a due obſervance of natural appearances furniſhed the firſt hints to intelligent painters how to proceed in this part of their art; for, by looking at a range of objects placed on lines parallel to each other, as rows of trees, &c. they could not but ſee them as converging together, and appearing nearer and nearer to each other in proportion to their remoteneſs: the ground, although really level, would ſeem as riſing upwards with a gentle aſcent; on the other hand, in viewing a level plane elevated on high, as a flat cieling, &c. they would ſee it as appearing to decline or ſink in going off from the eye.

Theſe, and ſuch like notices, may reaſonably be ſuppoſed the primary or principal guides by which artiſts were directed to make the imitations which they firſt produced of theſe kinds of appearances; and that illuſions of this ſort being publickly exhibited, induced geometricians,

who are not to be ſatisfied with any thing leſs than rigorous exactneſs, to examine ſtrictly into the cauſes of theſe effects, and the means of accurately inveſtigating perfect imitations of theſe illuſive appearances, and thereby Perſpective is become a ſyſtem of mathematical rules, the true guide and firm ſupport of the imitative arts.

The ancients made it a general and leading principle to conſider the objects, which they would repreſent, as being beyond a tranſparent plane, ſo placed between the eye and the objects, that all rays iſſuing from them to the eye ſhould paſs through the ſame, and impreſs thereon ſuch images as would have the ſame effect to an eye placed in the proper point for viewing them, as the real objects themſelves would produce when ſeen in their natural ſtate; and the moderns have aſſented to this poſition of the ancients; for, ſeeing that the image or repreſentation, thus imprinted on that plane, ſends to the eye the ſame rays as the real object itſelf would do, it is manifeſt, from all that has been diſcovered in optics, that no other ſenſation can be produced, but that ſimilar effects will be wrought upon the ſpectator; and that, if the variations of magnitude and poſition be duly aſſiſted by the art of colouring, the illuſion will be compleat: whence the art of Perſpective conſiſts abſolutely in a geometrical determination of the points in which the ſeveral rays cut the ſaid tranſparent plane in proceeding from each point of the given objects to the eye. In ſhort, a perſpective repreſentation is no other than a projection of objects, due regard being had to the poſition or place of the eye.

This fundamental principle of the ancients may be yet more generally extended; for it is not neceſſary to imagine the objects beyond a tranſparent plane, nor even that the plane ſhould be tranſparent; for the objects may be conſidered and treated as being between the eye and the plane on which they are to be repreſented,

and the visual rays, by which the objects are perceived, may be produced till they meet the plane which is to exhibit their images, and thereby mark out other points analogous to those of the original objects from whence they proceeded, which several rays will form on that plane, when thus posited, correspondent images, which will, in like manner, shew the perspective representations of the given objects; but the other way of applying this principle is most generally used, and is therefore particularly alluded to in def. I. page 17. nevertheless the geometrician is at liberty to have recourse to either, as shall be most convenient.

VITRUVIUS, in his Architecture, book VII. chapter I. has preserved some of the Perspective of the ancient Greeks; he observes, that one AGATARCHUS, having been instructed by ESCHYLUS how to draw theatrical decorations, was the first who wrote upon the subject; that Agatarchus taught his art to DEMOCRITUS and ANAXAGORAS, and that those two, who were geometers, had also wrote about it. Vitruvius adds, that they shewed how to draw lines from a point in a certain place, so as shall assuredly represent buildings in a decoration, in such manner, that some of them shall appear to project or advance towards the eye, while others at the same time seem to fall back and retire.

In this manner does the Roman architect explain the performances of Democritus and Anaxagoras; we need only be barely initiated in the rudiments of optics to enable us for distinguishing the resemblance which the principles of the ancient Perspective have to those of our own; for the point in a certain place, is that we call the point of sight, or place of the eye, which determines the position of almost all the lineaments of the object.

This is all we have left concerning the Perspective of the ancients, as far as I can find; so that we may, at least, fairly account the mo-

derns, the second inventors of this art, which took its rise among us in those glorious days of painting, the latter end of the fifteenth, or the beginning of the sixteenth century.

Two artists, who were also good geometricians, ALBERT DURER in Germany, and PIETRO DEL BORGO SAN STEPHANO in Italy, gave rules for putting objects in perspective. Durer did it mechanically by the help of a machine, whose construction and use are founded upon the ancient principle before-mentioned. Borgo, who was a little before Durer, wrote three books upon the subject, which Ignatio Dante highly extols, but they are lost to us.

After this author DANIEL BARBARO, patriarch of Aquileia, published a treatise on this art, in 1579.

BALTHAZAR PERUZZI, of Sienna, was happy in clearing Perspective of many incumbrances which it laboured under in Italy, and gave it a degree of elegance, by introducing the use of points different from those before made use of, as known to us, which are called POINTS OF DISTANCE: this author has been exactly followed, even to a degree of servility, by VIGNOLA, the famous Italian architect, in his Treatise of Perspective. SIRIGATTI made use of Vignola in the same manner; and ANDREA POZZO, so lately as the year 1700, indulged himself in a free use of the same liberty.

GUIDO UBALDI MARQUIS DE MONTE, in a folio treatise, printed at Pesaro in 1600, considered Perspective in a more scientific view than any of the former; he appears as being the first whose ideas tended towards rendering the principles of Perspective universal, and greatly improved the art, by advancing this very prolific principle, viz. that all lines, parallel to one another and to the horizon, although inclined to the table, or picture as it is now called, would constantly converge towards a point in the horizontal line, and that

the ſaid point is that where the ſaid horizontal line is interſected by the line drawn from the eye parallel to the afore-mentioned lines.

Ubaldi might indeed have made his principle more general, by ſhewing, that all lines parallel to one another, though not parallel to the horizon, do meet in one certain point of the picture, namely, in that where it is cut by that particular parallel ray which is drawn from the eye; and there are conditions which require this augmentation to what he advanced.

But what Ubaldi has given us may be made to ſuffice in the ordinary caſes of Perſpective, where ſuch objects moſt generally preſent themſelves as are terminated by lines perpendicular or parallel to the horizon; for it is plain, that the apparent concourſe of all lines perpendicular to the plane of the picture, being in the principal point, is only a particular caſe of Ubaldi's principle; for the ſaid principal point is no other than that in which the picture is interſected by the perpendicular drawn to the eye. In like manner, lines inclined 45 degrees to the plane of the picture, meet in that point of the horizontal line in which it is cut by a line, drawn from the eye under an angle of 45 degrees. All parallels, inclined 30 degrees to the plane of the picture, will appear to concur in that point thereof, in which a line, drawn from the eye under the like angle, meets the ſame; and the like of others. So that it would be no difficult matter to ſolve the general and fundamental problem of all Perſpective after 25 different ways, as Ubaldi has done, but the ſame may be performed by other methods innumerable. As for the reſt of Ubaldi's work, it has the fault common to others of that time of day, and what he has there expanded, through a multitude of propoſitions, might have been elegantly comprehended in the compaſs of a few pages.

With reſpect to what has been tranſmitted to us by Marolois, the Jeſuit, and others, whom we call the writers on the old Perſpective, in contradiſtinction to the ſyſtem we here endeavour to explain, it ſeems ſufficiently manifeſt, that the works of Peruzzi and Ubaldi have been the general ſtore-houſe to which thoſe ſeveral writers have had recourſe for the principles they make uſe of; we ſhall therefore paſs over, for the preſent, the numerous writings which have appeared on this part of the mathematicks, and not make any farther excurſions on a ſubject, whoſe greateſt difficulties are within the reach of a moderate geometrician, but proceed to ſhew what were the next advances made towards bringing it to a ſtate of perfection, and into general uſe among artiſts, and others deſirous of being acquainted with this uſeful and pleaſing art of deception.

The celebrated geometrician, DR. BROOK TAYLOR, F. R. S. obſerving how confined, intricate and ungeometrical, the principles of Perſpective were in his time, how few, ſimple and univerſal they in reality might be, deſirous of eſtabliſhing new principles of ſimple conſtruction, univerſal in application, and ſupported by geometrical demonſtrations, condeſcended to write on this ſubject, and publiſhed a ſmall tract, under the title of LINEAR PERSPECTIVE, in the year 1715.

In this little piece the doctor made prodigious advances towards bringing this art to its ultimate degree of perfection. He juſtly obſerved, that all planes, conſidered as ſuch, are alike in geometry, and ſhould in like manner be applied to Perſpective; that there are no excluſive honours due to the ground plane, nor any particular magic in the horizontal plane, nor conſequently in their correſpondent lines or points. He relieved us from the contracted limits in which our conceptions of this ſubject were inſcribed, enlarged and extended our ideas to univerſality itſelf, taught us the true uſe of vaniſhing

planes, lines and points, in all ſituations in which they can poſſibly be conceived, whether parallel, perpendicular, or any how inclining to the picture, or to the original objects, &c.

But finding that many objected againſt it on account of its not being ſufficiently eaſy to be generally underſtood; he therefore, in the year 1719, publiſhed a ſecond ſmall treatiſe, called NEW PRINCIPLES OF LINEAR PERSPECTIVE.

But this likewiſe, on account of the mathematical dreſs and brevity of expreſſion in which it was delivered, was not ſo generally careſſed as it deſerved to be, nor the principles it contained ſo much examined as he wiſhed; wherefore he deſigned the publick another treatiſe, propoſing therein to ſet forth thoſe principles in another light, ſuch that their preheminence above all others then in uſe might be more readily perceived, and the whole better adapted to the conceptions of young artiſts applying themſelves to Perſpective for works of deſign; but his death happening before the completion of thoſe intentions, the world was thereby deprived of the advantage of ſuch a perfect piece as might reaſonably be expected from his great abilities, and this method of Perſpective remained a knowledge enjoyed by few, and thoſe chiefly ſuch in whoſe hands it was not of the greateſt utility to the publick, through their not being of employments which required ſkill in thoſe principles, for making the proper applications of them required in the arts of deſign.

In the year 1738, JOHN HAMILTON, Eſq; F. R. S. publiſhed two volumes, folio, under the title of STEREOGRAPHY, or a compleat body of Perſpective in all its branches, the projections of ſhadows, reflexions by poliſhed planes, &c. in ſeven books. This learned and ingenious author hath very copiouſly treated the ſubject of Perſpective in a ſtrict mathematical way, with the aſſiſtance of Dr. Taylor's principles, whom he acknowledges to have made, in a few page

only, more real advances towards perfecting the ſcience of Perſpective, than all the writers who went before him. In this treatiſe, and alſo in thoſe wrote by MOXON, POZZO, MAROLOIS, LEYBOURN, and many others, he may ſee his miſtake, who aſſerts that it hath not been ſhewn how to find the perſpectives upon concave or convex ſurfaces, nor upon a figure of ſeveral faces, much leſs by reflexion or refraction, &c. * But this treaſury of mathematical projection is little attended to, as being too difficult and diffuſive for general uſe among artiſts, notwithſtanding it is worthy of being conſulted occaſionally in ſome extraordinary caſes that may chance to occur, and has been conſidered in what is here offered the reader.

In the year 1754, Mr. Kirby, deſigner in perſpective to their preſent MAJESTIES, publiſhed a treatiſe in quarto, titled Dr. Brook Taylor's Method of Perſpective made eaſy, &c. and in the year 1755, a ſecond edition thereof; which D. Fournier thought worthy for him to make free uſe of in his publication in the year 1764.

In 1761, he publiſhed the Perſpective of Architecture, a large and elegant work in folio, containing two rules of univerſal application; improvements in the doctrine of ſhadows, the deſcription and uſe of a new and very uſeful inſtrument, called the architectonic ſector; and the preſent year 1765 produces the third edition of that firſt publiſhed in 1754,

This ingenious author, by attentively examining and applying Dr. Taylor's new principles of Perſpective to practice, was gradually led to a diſcovery of their generality and facility in operation, ſaw how preferable and excellent they were in practical applications, how

* See Elements of Mathematicks, &c. together with a new Treatiſe of Perſpective, for the uſe of the Royal Academy at Woolwich. Printed for and ſold by J. Milan, Bookſeller, oppoſite the Admiralty, Charing-croſs, 1765.

ſimple and extenſive their conſtructions, what a vaſt confuſion of unneceſſary lines were thereby avoided, and how beneficial they would be if generally known to artiſts concerned in works of deſign; poſſeſſed with theſe and ſuch like conſiderations, he employed himſelf zealouſly to retrieve them from that ſtate of darkneſs in which their author's brevity of expreſſion and manner of writing had concealed them, and became the firſt among artiſts, who appeared in publick, to explain their true nature and uſe in adapting them ſuitably to the arts of deſign.

The encouragement he met with at his firſt communicating this deſign, how joyfully our artiſts in general embraced it, the ſucceſs which attended and overcame all the various oppoſitions he had to encounter in the proſecution of it, would come now to be explained, were I to continue the thread of my hiſtorical narrative beyond this period of time: but as theſe are things which would lead me to tranſmit an impartial account of ſome of this author's inventions, yet in manuſcript, as alſo to recite the exploded ſyſtems again revived, the controverſal writings produced thereupon, and other ſuch like attempts, made for continuing theſe new principles in their former obſcurity, which, being matters that I think not altogether proper to be explicitly handled in this co-temporary publication, I reſerve them to a future ſeaſon, and paſs over this part of its hiſtory in ſilence, for the ſake of preventing any undue ſuggeſtions which my perſonal acquaintance with Mr. Kirby might, perhaps, be the means of bringing upon the moſt fair and juſt account that can be given, ſhall therefore confine myſelf to only one general and well-atteſted fact, publickly declared to all, and which ſhould alſo, for the very ſame reaſon, have been here omitted, had it not appeared to me, that the juſtice due to the memory of thoſe ingenious authors, who have contributed to improve and exalt this art to perfection and ge-

neral utility, required my endeavours to reſcue them from the undeſerved cenſure lately paſſed upon them, of not having made the leaſt improvement*; as well as to vindicate ourſelves, by ſhewing that what we have advanced in favour of our author is not our own bold aſſertion only, but has the united ſuffrage of a body of artiſts, well qualified to judge deciſively in this matter, by whoſe order the following paragraph was inſerted in the ſeveral publick papers of the year 1754, viz.

ACADEMY of PAINTING and SCULPTURE, in St. Martin's-Lane. Jan. 24, 1754.

MR. KIRBY, author of a work, intituled, DR. BROOK TAYLOR'S METHOD OF PERSPECTIVE, MADE EASY, &c. has read three lectures (being the ſubſtance of his intended work) to the gentlemen of this ſociety, which appeared to them ſo clear, ſimple and extenſive, that, in order to do juſtice to ſo excellent a performance, they have unanimouſly given this their public approbation, and declared the ingenious author an honorary member of their body.

By order,

F. M. NEWTON, Secretary.

To conclude, I ſhall now only mention a few more of the many writings we have on this ſubject, viz.

HONDIUS, his perſpective inſtitutions were formerly held in great eſteem.

ALLEAUME's deſerves to be more known than it is, being well adapted to the purpoſes of artiſts.

FATHER DE CHALES's is remarkably neat.

S'GRAVESANDE's ESSAI DE PERSPECTIVE, publiſhed at Leiden in 1711, contains many new things, and is recommendable for practice.

* See page 311 of the Treatiſe before alluded to.

FATHER LAMY's contains ſome proper notices on the ſubject of painting.

M. DE LA CAILLE's performance deſerves to be noticed with reſpect.

THE NEW TREATISE, referred to in the preceeding note, is diſtinguiſhable for its extreme brevity and peculiar ſingularities, &c.

As to what I have here done, my whole deſign has been to lay before learners the method I have uſed in my private courſe of communicating a general knowledge of the theory on which the art of perſpective is grounded, ſo far as concerns one of my profeſſion. I do not pretend to a new treatiſe on perſpective, my only view is to illuſtrate and render the art eaſy to be underſtood, by a new manner of ſhewing the rationale of it, to facilitate the ſtudy, point out the true principles that ought to be applied by thoſe who would arrive at perfection in it, and know how to diſtinguiſh them from falſe ones, in an art where deception loſes all its beauty when ſo groſsly handled as not to bear a correſponding reſemblance to reality.

Page 11 ſhould begin thus,

Raiſe up the plane D Y, paſs the plane S H through the ſame, and make the plane B M paſs through the plane S H.

Whenever lines are mentioned, ſtraight lines are to be underſtood.

The references allude to the Elements of Euclid publiſhed by Profeſſor Simſon of Glaſgow, Octavo Edition 1762, thus (4. 1.) denotes the fourth propoſition of the firſt book, &c. And the doctrine of planes is principally taken from my predeceſſor's Elements of Geometry, Second Edition.

THE

THEORY OF PERSPECTIVE

DEMONSTRATED.

PART I.

OF THE DOCTRINE OF PLANES.

DEFINITIONS.

I.

A Line is perpendicular, or at right angles to a plane, when it makes right angles with every line meeting it in that plane.

ILLUSTRATION. Plate I. Fig. 1.

Make F in the plane F B coincide with F in the plane F N. Then, if the angles E D F, E D K, E D N, &c. are each a right angle, the line E D is perpendicular to the plane F N.

II.

A plane is perpendicular to a plane, when all lines drawn in one of the planes, perpendicularly to the common section of the two planes, are perpendicular to the other plane.

SAME FIGURE.

The lines B A, E D, G F, &c. being drawn in the plane B F perpendicularly to A F, the common ſection of the two planes B F, F N, if they are alſo perpendicular to the plane F N, the plane B F paſſing through thoſe lines, is perpendicular to the plane F N.

III.

The inclination of a line to a plane is the acute angle formed by that line, and another line drawn from the point, in which the firſt line meets the plane, to the point in which a perpendicular to the plane, drawn from any point of the firſt-mentioned line above the plane, meets the ſame plane.

SAME FIGURE.

Thus the acute angle F D G is the inclination of the line D G to the plane F N.

IV.

The inclination of a plane to a plane is the acute angle formed by two lines drawn from any the ſame point of their common ſection at right angles to it, one upon one plane, and the other upon the other plane.

SAME FIGURE.

Raiſe up the planes D Y, Z X, making W Y coincide with W y. Then the line D W, being the common ſection of the planes D Y, D Z, and R T perpendicular thereto, and R S alſo perpendicular thereto, at the ſame point R, the acute angle S R T is the inclination of the plane D Y to the plane D Z.

V.

Two planes are ſaid to have the ſame or a like inclination one to the other, which two other planes have to each other, when the ſaid angles of inclination are equal to one another.

VI.

Two planes, which, being either way produced, do not meet each other, are ſaid to be parallel one to the other.

VII.

A pyramid is a ſolid figure contained by planes that are conſtituted betwixt one plane and one point out of that plane, all meeting in that one point.

ILLUSTRATION. Plate II. Fig. 2.

Make B in the triangle L A B coincide with B in the plane L B, alſo make C coincide with C, and L with L, and B in the triangle B A E with B in the plane L B, then will the Figure thus formed repreſent a pyramid.

VIII.

The point A is the vertex of the pyramid. The plane L B the baſe thereof. The planes L A B, B A C, &c. the Sides thereof. And the line A H the perpendicular altitude or height thereof above the plane of its baſe L B.

THEOREM I.

One part of a line cannot be in a plane and another part thereof above it.

PLATE I. Fig. 1.

Make F in the plane F B coincide with F in the plane F N.

DEMONSTRATION.

For, if it be poſſible, let A D, part of the line A D G, be in the plane F N, and the part D G above the ſame, then A D being in the plane F N, it can be produced in that plane ſuppoſe to F; now the points D and G are in the plane B F, which paſſes through the

line A F, whence the line D G is in the plane B F, (7 def. 11.) wherefore the two lines A D G, A D F, are in the ſame plane, and have a common ſegment A D, which is impoſſible (cor. *1*. 1.)

Q. E. D.

THEOREM II.

Two lines, which cut one another, are in one plane, and three lines, which meet one another, are in one plane.

SAME FIGURE.

Make F coincide with F, and the planes B M, S H paſs through each other in the line H L, and let p k, r H be two lines cutting one another in C; I ſay thoſe two lines, as alſo the three lines C H, C k, H k, meeting each other, are in one plane.

DEMONSTRATION.

For neither can p C, part of p k, nor r C, part of r H, be in one plane, and the other parts C H, C k, out of that plane by the preceedent; wherefore the lines p k, r H are in one plane. And if you ſay that C P x, part of the triangle C H k, is in one plane, and the other part P H k x in another plane, then muſt C P be in one plane and P H in another, the ſame likewiſe of C x and x k, which is impoſſible; therefore the triangle C H k is in one plane. *Q. E. D.*

THEOREM III.

If two planes interſect each other, their common ſection is a ſtraight line.

SAME FIGURE.

Let H and L be the extremes of the common ſection of the two planes B M, S H, join thoſe extremes by drawing the line H L.

DEMONSTRATION.

Then is the line H L in both the ſaid planes; that line is therefore their common ſection by conſtruction, for if you ſay it is not in both the planes, then may two lines be drawn between the ſame extremes, one from L to H in the plane S H, and another from L to H in the plane B M; thus would two lines, bounded between the ſame extremes, include a ſpace, which is impoſſible, (axiom 10. 1.) wherefore H L muſt neceſſarily be in both the planes S H, B M, which is therefore their common ſection. *Q. E. D.*

THEOREM IV.

If one line be perpendicular to the common ſection of two other lines which interſect each other, it is alſo perpendicular to the plane which paſſes through the ſaid two interſecting lines.

ILLUSTRATION. Plate II. Fig. 3.

Make E in the plane E B C to coincide with E in the plane C D E F, and paſs the plane F B D through the plane E B C, alſo make the plane G B H paſs through both the ſaid planes, making H coincide with H.

DEMONSTRATION.

Let there be taken A C=A D=A E=A F, and draw the lines C D, C F, F E, E D alſo through A in the plane C D E F, draw the line G H any how, meeting the lines C F, D E, in G and H, and let A B be perpendicular to the two lines C E, D F at their common ſection A, and draw the lines B C, B F, B E, B H, and B D.

Now by conſtruction A C, A D, A E and A F are equal to each other, and the angle C A F=D A E, (15. 1.) whence C F=D E (4. 1.) and the angle F C A or G C A=D E A or H E A, ſo like-

wiſe G A C=H A E, and A C=A E; therefore is A G=A H and G C=H E (26. 1.)

Again, becauſe A B is perpendicular to the plane C D E F by hypotheſis, the triangles C A B, D A B, E A B, F A B, are each right angled at A, having their baſes equal each to each, and the perpendicular A B common to them all, whence their hypothenuſes will alſo be equal each to each, B C=B D=B E=B F; wherefore the triangles C B F, D B E, being mutually equilateral, the angle F C B or G C B=D E B or H E B, (8. 1.) and becauſe G C=H E and B C=B E, therefore is B G=B H.

But it was proved above, that A G=A H, and A B is common, whence the angle G A B=H A B, conſequently A B is perpendicular to the line G H (def. 10. 1.) and in the ſame manner it may be proved, that A B is perpendicular alſo to C E, F D, and all other lines whatever that can be drawn in the plane C D E F through the point A, on which it inſiſts; therefore A B is perpendicular to the plane paſſing through the lines C E, D F, &c. *Q. E. D.*

COROLLARY.

Hence it follows, that when a line, as A B, is at right angles to ſeveral lines, as A F, A E, &c. which it meets in the ſame point, as A, the lines which it ſo meets, are all in the ſame plane; for a line, as A K, drawn from A out of the plane C D E F, cannot be perpendicular to A B, becauſe the angle B A K will be leſs or greater than a right angle, or B A H, according as A K is drawn above or below the ſaid plane C D E F.

THEOREM V.

If in a given plane a line be drawn through any point thereof, and two other lines perpendicular thereto be alſo drawn from the ſaid

point, one of them in the given plane, and the other in any other plane, passing through the first mentioned line; I say, that if from the said point a line be drawn, in the plane passing through those two perpendiculars, at right angles to the first of them, it will be perpendicular to the given plane at the said point.

PLATE I. FIG. 1.

Raise up the plane D Y, pass the plane S H through the same, and the plane B M through the plane S H.

Let now B M be the given plane, C a point, and L H a line drawn through the same. C E, C R lines perpendicular to L H, the first in the given plane B M, and the other in another plane, as S H, passing through the first mentioned line L H. Also let D Y be a plane passing through both the perpendiculars C E, C R, and conceived as being produced. Now let C I be a line drawn from C, in the produced part of the plane D Y, at right angles to the first perpendicular C E; then will C I be also perpendicular to the given plane B M at the said point C.

DEMONSTRATION.

Because C H is perpendicular both to C E and C R, by hypothesis, it will also be perpendicular to C I, by the preceedent, whence C I being perpendicular both to C E and C H at the point of meeting, it is therefore perpendicular to the given plane B M, containing the said lines, by the preceedent. *Q. E. D.*

THEOREM VI.

Two lines, which are perpendicular to one and the same plane, are parallel to each other.

SAME FIGURE.

Make K in the triangle G F K coincide with K in the plane F N,

and let E D and G F be each perpendicular to the ſame plane F N, and in that plane draw the line F D, and perpendicular thereto draw D K=F G, alſo let there be drawn the lines F K, G K and G D.

DEMONSTRATION.

The triangles G F D, F D K, having G F=D K, by conſtruction, the ſide F D common, and the angle G F D=F D K, each being a right angle, will alſo have G D=F K, (4. 1.) whence the triangles G D K, G F K being mutually equilateral, the angle G D K or G F K is a right angle, (8. 1.) but the line E D is alſo perpendicular to D K, as well as the lines G D, F D; it is therefore in the ſame plane with them E D F G, conſequently the angles G F D, E D F being right angles, E D is therefore alſo parallel to F G. *Q. E. D.*

COROLLARY.

Hence it appears, that there cannot be drawn more than one perpendicular from the ſame point to one and the ſame plane, for all lines perpendicular to the ſame plane are parallel to each other, which lines, drawn from one and the ſame point, can not be.

THEOREM VII.

If there be two parallel lines, one of which is perpendicular to a plane, the other will alſo be perpendicular to that ſame plane.

SAME FIGURE.

The angles F D K, G D K being right angles, and G D in the ſame plane with the propoſed parallels G F, E D, the angle E D K is alſo a right angle, (4. 11.) as is alſo the angle E D F; therefore is E D perpendicular to the plane F D K. *Q. E. D.*

THEOREM VIII.

If a line be drawn from any point in one of two parallel lines to any point in the other, that line will be in the ſame plane with the ſaid parallels.

SAME FIGURE.

Let B X, E Q be two parallel lines in the plane B M, and take any point H in one of them, and any point C in the other, then will the line which joins the points H and C, be in the plane of the propoſed parallels.

DEMONSTRATION.

If you ſay it is not, ſuppoſe it above that plane as the line C u H, and in the plane of the parallels draw C H, then will the lines C u H, C H, be terminated by the ſame extremes, and include a ſpace, which is impoſſible, (ax. 10. 1.) whence the line joining the points C and H can not be above the plane of the propoſed parallels, in like manner it will appear that it can not be below the ſame; therefore it muſt be in the ſame plane with them. *Q. E. D.*

THEOREM IX.

If a line be perpendicular to a plane, any plane paſſing through that line, will be at right angles to the plane whereto the ſaid line is perpendicular.

SAME FIGURE.

Let E D be a line perpendicular to the plane F N, and B F a plane paſſing through that line, and it will alſo be perpendicular to the ſaid plane F N.

DEMONSTRATION.

From any point, as B, taken at pleaſure in the plane B F, draw

B A perpendicular to A F, the common section of the two planes B F, F N, then is the angle F A B = F D E, equal a right angle, (def. 3. 11.) whence B A is parallel to E D, and is therefore perpendicular to the plane F N, by the foregoing theorem 7. In like manner it may be proved, that every line which can be drawn in the plane B F perpendicularly to A F, the common section of the planes, will be also perpendicular to the plane F N, therefore the plane B F, passing through those lines, will itself be also perpendicular to the plane F N. *Q. E. D.*

COROLLARY I.

Hence it follows, that the plane F N, according to the sense of the definition, is perpendicular to the plane B F; for let D K be a line drawn in the plane F N, perpendicular to A F, the common section of both planes, at the point D, and it will also be perpendicular to E D; and therefore the plane F N, in which it was so drawn, is also perpendicular to the plane B F, passing through the said line E D.

COROLLARY II.

Hence it also follows, that a line, drawn at right angles to one of two perpendicular planes from any point of their common section, will be in that other plane. For let the two perpendicular planes be F N, B F, their common section A F, and D a point in the same; also let D E be at right angles to the plane F N, then must D E be in the plane B F, for no other line can be drawn from the point D at right angles to the plane F N, as appears by the corollary to the foregoing theorem 6.

THEOREM X.

Planes, to which one and the same line is perpendicular, are parallel to each other.

SAME FIGURE.

Let E C be perpendicular to the plane B F, and alſo to the plane S H, then are thoſe planes parallel.

DEMONSTRATION.

From any point, as D, in the plane B F, draw D R parallel to E C, and it will alſo be perpendicular to both the planes B F, S H, by theorem 7. Let the lines E D, C R be drawn, now ſeeing the angles at E, C, R and D, are each right ones, (def. 1.) the figure E C R D is a rectangled parallelogram, by reaſon its ſides E D, C R, are in the ſame plane D Y, with the parallels E C, D R, conſequently E C = D R, the ſame way it can be proved, that all other perpendiculars, terminated by thoſe two planes B F, S H, are alſo equal to each other; therefore the ſaid planes are parallel. *Q. E. D.*

COROLLARY.

Hence it appears, that all lines, which are perpendicular to one of two parallel planes, are alſo perpendicular to the other.

SCHOLIUM.

From the two laſt theorems, the ſenſe and propriety of the two definitions of perpendicular and parallel planes appear manifeſt.

THEOREM XI.

Lines which are parallel to one and the ſame line, though not in the ſame plane with them, are parallel to each other.

SAME FIGURE.

Paſs the plane S H through the plane D Y, alſo paſs the plane B M

through the planes S H and Z X, and let B G, X M, be each parallel to Z Y, then are they alſo parallel to each other.

DEMONSTRATION.

Draw E W, W Q, each perpendicular to Z W, then will W Q be perpendicular to the plane paſſing through E W Q, and E B, Q X, will alſo be perpendicular to the ſame plane, whence they are alſo parallel to each other; therefore the lines B G, X M, are parallel one to the other. *Q. E. D.*

THEOREM XII.

If two lines, meeting each other in one plane, be reſpectively parallel to two other lines alſo meeting each other in ſome other plane, then will the angle, formed by the two lines meeting in the firſt-mentioned plane, be equal to that formed by the meeting of the other two lines in the other plane.

SAME FIGURE.

Let the lines L C, C R, in the plane S H, be reſpectively parallel to the lines G E, E D, in the plane B F, then will the angle L C R be equal to the angle G E D.

DEMONSTRATION.

Take C L, C R, G E, E D, all equal to one another, and let C E, R D, D G, L R, G L, be drawn, then C R and D E, being equal and parallel to each other, as alſo L C equal and parallel to G E by hypotheſis and conſtruction, G L and R D are both equal and parallel to C E, conſequently equal and parallel to each other, (33. 1.) whence D G = L R, and the triangles L C R, G E D, be-

ing mutually equilateral, the angle L C R is therefore equal to the angle G E D (8. 1.) *Q. E. D.*

THEOREM XIII.

If two lines, meeting each other in one plane, be reſpectively parallel to two other lines, meeting each other in another plane, then the two planes, paſſing through thoſe lines, will alſo be parallel to each other.

SAME FIGURE.

Let the lines E D, E G, in the plane B F, be reſpectively parallel to the lines U c, U O, in the plane S H, then are the planes B F, S H, paſſing through thoſe lines, parallel to each other.

DEMONSTRATION.

Draw E C perpendicular to the plane B F, meeting the plane S H in C, in which plane let there be alſo drawn C r, r ſ, reſpectively parallel to U O, U c, then are C r, r ſ, reſpectively parallel to E G, E D, by theorem 11, and becauſe C E G, C E D, are right angles by conſtruction, E C L, E C R, are likewiſe right angles; whence E C, being perpendicular to the plane S H, by theorem 4, and alſo to the plane B F, by conſtruction, the two planes B F, S H, are therefore parallel one to the other, by theorem 10. *Q. E. D.*

THEOREM XIV.

If two parallel planes are cut by a third plane, the ſections made thereby are parallel to each other.

SAME FIGURE.

Let the parallel planes B F, S H, be cut by the plane D Y, then are the ſections E D, C R, parallel one to the other.

DEMONSTRATION.

Draw E L and D S parallel to each other, alſo draw E C, D R, perpendicular to the plane S H, and draw C L, S R, then E L being parallel to D S, by conſtruction, and E C parallel to D R, by theorem 6, the angle C L E = R S D, by theorem 12, but the angle E C L = D R S, as being each a right angle, and E C = D R; whence E L is both equal and parallel to D S, therefore E D is alſo parallel to C R. *Q. E. D.*

COROLLARY.

Hence it appears, that lines parallel to each other, and terminated by the ſame parallel planes, are alſo equal to each other.

THEOREM XV.

If, from the extremes of a line paſſing through a given plane, be drawn two lines, each meeting the ſaid plane at right angles, the line, which joins the two points where thoſe two perpendicular lines meet the ſaid plane, will be cut by the firſt-mentioned line, in that point where it paſſes through the ſaid plane, and be thereby divided into two ſuch parts, as bear the ſame ratio to each other, which the two perpendiculars, meeting the plane, have one to the other.

SAME FIGURE.

Let n m be the line cutting the given plane B M in the point C, and from n and m, the extremes thereof, draw the perpendiculars n r, m H, meeting the plane S H in the points r and H, and draw the line H r; then will r C : C H :: r n : H m.

DEMONSTRATION.

Produce n r as here to ſ, then ſeeing n ſ, m H, are each perpendicular to the plane B M, by hypotheſis, they are parallel to each

other by theorem 6, whence the lines n m, H r, being both in the ſame plane with thoſe parallels, and terminated by them, as they are not parallel to each other, they will meet or interſect each other, and thereby make the alternate angles equal one to the other, that is, r n C = H m C, as alſo r C n = H C m; whence the triangles n r C, m H C, being ſimilar, it will therefore be r C : C H :: n r : m H. *Q. E. D.*

COROLLARY.

Hence it follows, that if, in the plane B M, there be drawn the lines r f, H X, parallel to each other, and in them be taken r p = n r, and H k = H m, then will the line p k, joining the points p and k, interſect H r in the ſelf ſame point in which it was before interſected by the line n m. For if h be conceived as the interſection point of p k with H r, the triangles p r h, H h k, will be equiangular; whence we have r h : h H :: r p : H k,

and n r : H m :: r C : C H,

but r p = n r, and H k = H m,

wherefore n r : H m :: r C : C H; therefore, ſeeing r H is divided into one and the ſame ratio by both the points C and h, thoſe two points muſt neceſſarily coincide together, or vaniſh into one point only.

THEOREM XVI.

If two planes, interſecting each other, be both perpendicular to a third plane, the common ſection of thoſe two planes will be alſo perpendicular to the ſaid third plane.

SAME FIGURE.

Let the planes D Y, S H, be each perpendicular to the plane A Z, interſecting each other in the line C R, then is their common ſection C R perpendicular to the plane A Z.

DEMONSTRATION.

From R, one extreme of the common section, draw a line, as R C, perpendicular to the plane A Z, then this perpendicular, being in both the planes D Y, S H, by cor. 2 to theorem 9, it must necessarily be their common section, which is therefore perpendicular to the plane A Z, by construction. *Q. E. D.*

PART II.

OF THE DOCTRINE OF PLANES APPLIED TO THE TRUE PRINCIPLES OF PERSPECTIVE.

DEFINITIONS.

I.

PERSPECTIVE is that part of mathematical projection which gives rules for describing, upon any given plane, the representations of any given objects, so as to exhibit thereon the exact forms, magnitudes and positions thereof, such as they would be seen to have upon that plane, by an eye fixed in one certain point, and viewing them from thence through the said given plane, supposing it to be perfectly transparent, or as glass, &c.

ILLUSTRATION. Plate III. Fig. 4.

Raise up the plane D Y, and pass the plane S H through the same, in the line C P, then bring R, in the plane F G, to coincide with R in the plane F M, raise up the plane M X, and pass the plane G X through both the planes S H and M X.

Suppose now the plane S H transparent, and A B a line in the plane F M, seen by an eye fixed at E, and viewing it through the plane S H, then will A and B, the extremes of that line, appear on the plane S H, in the points a and b, where the lines A E, B E, drawn from, or issuing thence to the eye at E, cut or intersect the said plane S H; and all other lines which can be drawn, or conceived to issue from any other of the points in the line A B to the eye at E, will, by their intersections with the plane S H, mark out other intermediate points between a and b, and the whole line A B

will be thereby depicted, on the plane S H, by the line a b, which is therefore the true description or representation thereof.

SCHOLIUM.

Hence it appears, that the given plane, on which the representations of the given objects are to be drawn, must be so situated *in respect of* the given objects and the point in which the eye is placed, that all lines, which can be drawn or conceived to issue from them to the eye, may pass through the said plane; for, seeing the representations thereof are determined upon that plane by the intersections made thereon, by lines passing through it in proceeding from them to the eye, it is plain there can not be any representations formed on that plane of objects which are so posited, that the lines issuing from them to the eye, do not cut the plane on which they are required to be described; whence the necessity of the above-mentioned position of the said given plane appears manifest.

II.

The plane S H, upon which the given objects are represented, is, in mathematical terms, called the plane of projection, but here we term it the PICTURE.

III.

The plane, which contains the objects given to be described on the picture, is called the ORIGINAL PLANE.

Thus the plane F M is here the original plane.

IV.

The point, in which the eye is fixed for viewing the given objects, or their representations on the picture, is called the place of the eye, or POINT OF SIGHT.

Thus E is the point of sight.

SCHOLIUM.

The place of the eye, or point of ſight, ought to be juſtly aſcertained, by having due regard to the ſize, ſituation, &c. of the picture and original plane; for an injudicious determination in this reſpect, produces ill effects in the repreſentations formed on the picture, as appears from a recent inſtance hereof in a late production, in which the author has made ample diſplay of inconſiſtent repreſentations, formed by tranſgreſſing the rules requiſite to be obſerved on this occaſion. How faults of this kind are to be avoided, will be ſhewn in the annotations or general comment hereto annexed.

V.

A line drawn from the eye, perpendicular to the picture, and meeting the ſame, is called the DISTANCE, or AXIS OF THE EYE, or diſtance of the picture.

Thus E C is the diſtance or axis of the eye, or diſtance of the picture.

VI.

The point, in which the axis of the eye meets the picture, is the CENTER OF THE PICTURE.

Thus C is the center of the picture S H.

VII.

The deſcriptions made upon the picture of any original objects, whether they be points, lines, ſurfaces or ſolids, are called the perſpective repreſentations or IMAGES thereof.

Thus the line a b is the perſpective repreſentation or image of the given line A B.

VIII.

A plane, paſſing through the point of ſight, parallel to any original plane, is called the VANISHING PLANE of that original plane.

Thus the plane G X, being parallel to the original plane F M, and paſſing through E, the point of ſight, is the vaniſhing plane of the ſaid plane F M.

IX.

A plane, paſſing through the point of ſight, parallel to the picture, is called the DIRECTING PLANE.

Thus the plane F G, being parallel to the picture S H, and paſſing through E, the point of ſight, is the directing plane.

X.

A plane, paſſing through the axis of the eye at right angles to the original plane, is the VERTICAL PLANE of the ſaid original plane.

Thus D Y is the vertical plane of the original plane F M.

XI.

The line, in which the directing plane cuts the original plane, is called the DIRECTING LINE of that original plane.

Thus F R is the directing line of the original plane F M.

XII.

The line, in which a vaniſhing plane cuts the picture, is called the VANISHING LINE of the original plane correſponding to that vaniſhing plane.

Thus H L is the vaniſhing line of the original plane F M.

XIII.

The line, in which a vaniſhing plane cuts the directing plane, is called the EYE'S PARALLEL.

Thus I G is the parallel of the eye when placed in E.

XIV.

The line, in which the vertical plane interſects the directing plane, is called the DIRECTOR, or HEIGHT OF THE EYE.

Thus E D is the director or height of the eye when fixed in E.

XV.

The line, in which the picture interſects the original plane, is called the INTERSECTING LINE of that plane.

Thus S P is the interſecting line of the original plane F M.

XVI.

The line, in which the picture interſects the vertical plane, is called the VERTICAL LINE of the original plane.

Thus C P is the vertical line of the original plane F M.

XVII.

The point, in which an original line, produced when neceſſary, cuts the interſecting line, is called the INTERSECTING POINT of that original line.

Thus P is the interſecting point of the original line A B, and S that of the original line M N.

XVIII.

The point, in which an original line, produced if neceſſary, cuts the directing line, is called the DIRECTING POINT of that original line.

Thus D is here the directing point of A B, and alſo of M N.

XIX.

The point, in which a line, drawn from the point of ſight, parallel to any original line, cuts the picture, is called the VANISHING POINT of that original line.

Thus E C, being drawn parallel to A B, the point C is the vaniſhing point of A B; and E V, being parallel to M N, the point V is the vaniſhing point of that line M N.

XX.

The point, in which the vertical line cuts a vaniſhing line, is called the CENTER OF THAT VANISHING LINE.

Thus C is the center of the vaniſhing line H L, the ſame here with that of the picture.

XXI.

The diſtance between the point of ſight and the center of a vaniſhing line, is called the DISTANCE OF THAT VANISHING LINE.

Thus E C is the diſtance of the vaniſhing line H L.

XXII.

The diſtance between the point of ſight, and the point in which a line drawn from thence, parallel to any original line, cuts the picture, is called the DISTANCE OF THE VANISHING POINT of that original line.

Thus E C is the diſtance of C, the vaniſhing point of A B, and E V, the diſtance of V, the vaniſhing point of M N.

XXIII.

When the image on the picture is a point, the line which produces it in paſſing from the original to the eye, is called a RAY, or OPTIC RAY, &c.

Thus A E, producing the image a, is called a ray, as is alſo B E, which produces the image b.

XXIV.

When the image is a line, the ſeveral rays which produce it are together called a PLANE OF RAYS.

Thus all the rays, that can be conceived to iſſue from each of the points in A B, meeting together in E, conſtitute a plane of rays.

XXV.

When the original is a circle, then the ſeveral rays, which produce its image on the picture, are called a CONE OF RAYS.

XXVI.

And when the original is a right-lined ſurface or ſolid, the rays,

iſſuing from the ſeveral points thereof, are called a PYRAMID OF RAYS.

ILLUSTRATION. Plate IV. Fig. 5.

Raiſe up the plane D C, and make the plane O P paſs through the ſame, then raiſe up the plane S T, and bring D, in the plane P D, to coincide with D in the plane D K, and paſs the plane L B through both the planes O P, S T, alſo bring D, in the plane D Y, to coincide with D, and paſs the plane M N through both the planes O P, S T.

Then the three planes of rays B E C, C E G, B E G, taken together, form a pyramid of rays, and ſo of the others, as exhibited in the figure.

THEOREM I.

The parallel of the eye, interſecting, vaniſhing, and directing lines, belonging all to the ſame original plane, are parallel to each other.

PLATE III. Fig. 4.

Make the picture S H paſs through the vertical plane D Y in the line C P, and paſs the plane G X through both the planes S H and M X.

Now let F M be the original plane, I G the parallel of the eye, S P the interſecting, H L the vaniſhing, and F R the directing lines thereof; I ſay, thoſe ſeveral lines are parallel one to the other.

DEMONSTRATION.

Becauſe the picture and directing plane are parallel, (def. IX.) the vaniſhing and original planes alſo parallel, (def. VIII.) the ſections made by thoſe planes are parallel; (theorem XIV. part I.) therefore the lines I G, S P, H L and F R, are parallel to each other.

Q. E. D.

COROLLARY.

Hence it follows, that any one of thoſe lines, and any point in another of them, being given, that other line may thence be determined; for a line, drawn through that point, parallel to the given line, will be the line required. But it does not follow, as an axiom, that theſe two lines will be in the ſame plane, for, before that can be rightly aſſerted, the interpoſition of another plane, diſtinct from the former, muſt be introduced.

THEOREM II.

The vertical plane is perpendicular to the picture, the vaniſhing, directing and original planes, and alſo to the parallel of the eye, the interſecting, vaniſhing, and directing lines of that ſame original plane.

SAME FIGURE.

Let D Y be the vertical plane, and the other mentioned planes and lines the ſame as in the preceedent.

DEMONSTRATION.

The line E C is perpendicular to the picture, (def. V.) and the vertical plane D Y paſſes through that line, (def. X.) the vertical plane is therefore alſo perpendicular to the picture, (theorem IX. part I.) and likewiſe to the directing plane, which is parallel thereto. (def. IX.) the ſaid vertical plane is alſo perpendicular to the original plane, (def. X.) whence it is alſo perpendicular to the vaniſhing plane, that being parallel thereto; (def. VIII.) wherefore the ſaid vertical plane, being perpendicular to each of the four above-mentioned planes, it is therefore perpendicular to the common ſections I G, H L, S P, F R. *Q. E. D.*

THEOREM III.

The director is perpendicular both to the directing line and the parallel of the eye. And the vertical line is perpendicular both to the interfecting and vaniſhing lines.

SAME FIGURE.

Let E D be the director, C P the vertical line, and the reſt as before.

DEMONSTRATION.

The lines I G, H L, S P, F R, being all perpendicular to the plane D Y by the preceedent, they are therefore perpendicular to all lines meeting them in that plane. (def. I. part I.) Now, ſeeing E C, C P, P D, D E, are in the plane D Y, therefore is E D perpendicular to F R and I G; alſo C P perpendicular to H L and S P, conſequently it is as the theorem enounces. *Q. E. D.*

THEOREM IV.

Original planes, parallel to the picture, have neither vaniſhing, interfecting, nor directing lines, neither have the lines, ſituated in thoſe planes, any of thoſe correſponding points.

SAME FIGURE.

Let M X be an original plane parallel to the picture S H, and Z X an original line ſituated in that plane; I ſay, the plane M X hath neither vaniſhing, interfecting, or directing lines, nor hath the line Z X any of them points.

DEMONSTRATION.

For the original plane M X, being parallel to the picture, can not cut it to determine the interfecting line as is required, (def. XV.) the

vaniſhing plane F G being alſo, in this caſe, parallel to the picture S H, can not determine a vaniſhing line to the original plane M X, (def. XII.) for the like reaſon there can be no directing line, the directing plane F G being here parallel to the original plane M X, as well as to the picture S H.

Again, ſeeing the plane G X drawn through the eye at E and the original line Z X, cuts the picture in H L, and the directing plane in I G, thoſe two lines are parallel, (theorem XIII. part I.) they are alſo parallel to Z X, ſince a plane M X may be drawn through that line parallel to the plane S H; whence I G is the line which ought to produce the vaniſhing point of the line Z X, (def. XIX.) but, being parallel to the picture, it can not. The original line Z X, being alſo parallel both to the picture and directing plane, can cut neither of them; therefore the plane M X and the line Z X, are deſtitute of either vaniſhing, interſecting, or directing lines, or points. *Q. E. D.*

COROLLARY I.

If an original plane M X, parallel to the picture, cut any other plane whatever, as F M, their common ſection A M will be parallel to the vaniſhing, interſecting, and directing lines of the ſaid plane F M.

For the original plane M X, being parallel to the picture and directing plane F G, the ſections A M, S P, F R, made by thoſe three planes, with the plane F M, are parallel. (theorem XIII. part I.) Now S P is the interſecting line, F R the directing line of that plane, H L the vaniſhing line thereof, which is alſo parallel thereto; therefore A M is likewiſe parallel to that vaniſhing line, (theorem X. part I.)

COROLLARY II.

Hence it appears, that the image of an original line, parallel to the picture, is parallel to the ſaid original.

SAME FIGURE.

Let Z X be an original line parallel to the picture S H, and let the plane G X paſs through the point of ſight, and the original line Z X, cutting the picture S H, in the line H L, draw the lines X E, Z E, interſecting the picture in the points W and V; then is the line W V the image of Z X, and they are parallel to each other, by what was proved above.

COROLLARY III.

All parallel original lines, which are parallel to the picture, have parallel images, for thoſe images are parallel to their correſponding originals, by the preceedent corollary.

COROLLARY IV.

Hence it follows, that whatever angle is formed by the meeting of two original lines, which are parallel to the picture, an angle equal thereto will be formed on the picture by the meeting of their correſpondent images.

SAME FIGURE.

Make the planes D Y, D Z, both paſs through the picture S H in the lines C P, V S; raiſe up the plane M X, ſo that Y coincide with Y, and make the plane G X form one continued plane with the plane F E, and let A Y, Y M, be two original lines parallel to the picture, making the angle A Y M, then is the angle **a** C m, formed on the picture by their correſpondent images C**a**, C m, equal thereto.

For thoſe images are reſpectively parallel to their originals, (by the preceedent corollary) therefore the angle a C m = A Y M. (theorem XII. part I.) And, univerſally, all lines in an original plane, which is parallel to the picture, are themſelves parallel to the picture, and alſo to their reſpective images.

THEOREM V.

The image of an original line, which is not parallel to the picture, produced both ways, will paſs through both the interſecting and vaniſhing points of the ſaid original line.

SAME FIGURE.

Let now the plane D Y form one continued plane with the plane D M, paſs the plane D Z through the picture S H in the line V S, making D coincide with D and M with M, and paſs the plane G X through the picture. Let now M N be an original line in the original plane F M; S the interſecting, and V the vaniſhing point thereof.

DEMONSTRATION.

Becauſe the lines E V, M N, are parallel, (def. XIX.) all lines that can be drawn from E to the line M N, will be in the ſame plane as the lines E V and M N are in. (7. 11.) Now, ſeeing the points S and V are in this plane, and alſo in the picture, the line V S is the common ſection of thoſe two planes, but n m, the image of N M, is a part of that ſection; therefore n m, produced both ways, will paſs through S and V, the interſecting and vaniſhing points of the original line. *Q. E. D.*

COROLLARY I.

The directing point D can have no image on the picture, by reaſon the line E D, which ought to cut the picture in order to form

an image, is in the directing plane F E; it is therefore parallel to the picture, consequently can not interfect it.

COROLLARY II.

The images of all the points, that can be conceived between the interfecting point S and the extreme point M, how far foever it be produced on from thence, will fall on the picture between the interfecting point S and the vanifhing point V; for, wherever the point M be fo taken, the angle E M S will be equal to the angle M E V, as being the alternate angles of two parallel lines; wherefore the points m and V can never coincide, nor can the point N be fo taken, but that its image will always be between S and V, the angle E N S being, for the like reafon, always equal to the angle N E V. Now, when N comes into S, its image n then alfo comes into the interfecting point S; in like manner, if M be fuppofed at an infinite diftance, fo that the angle M E V entirely vanifh, and the line E M coincide with E V, then will its image m vanifh into the point V. Hence appears the propriety of calling this a vanifhing point, as alfo why the line H L, paffing through that point, is termed a vanifhing line, and the plane G X, paffing through that line, a vanifhing plane, &c.

COROLLARY III.

If the original line itfelf paffes through its own vanifhing point, its image is then contracted into that point, and the line, in this cafe, may be faid to vanifh. Thus, fuppofe V the vanifhing point of N M, and let N M be conceived as moving parallel to itfelf along the lines N K, M Z, till, being in the pofition or line Z K, its produced part N S or K V paffes through the point V, then is that point only the image of the line N M when in that pofition, the images n, m, both now coinciding in V.

COROLLARY IV.

Hence it follows, that if an original line be produced and paſs through the point of ſight, the vaniſhing and interſecting points thereof will coincide, and its directing point will be the point of ſight; whence any point in the picture may be the image of any point in the original line paſſing through the eye and that point of the picture, or may be taken as the vaniſhing point of that line, or as the image of its interſection with all planes or lines whatſoever, which it cuts; wherefore, a point being given on the picture, its original can not be thence determined, unleſs the poſition of that original, with reſpect to ſome other known point, line or plane, be alſo given.

THEOREM VI.

Original lines, which are parallel to each other, but not to the picture, have the ſame vaniſhing point, and their images produced all meet in that ſame point.

SAME FIGURE.

Raiſe up the plane D Y, and paſs the picture S H through the ſame; bring R to coincide with R, and make the plane G X paſs through the picture. Let now O Q be parallel to A B, then is C their vaniſhing point, and their images a b, o q, produced, will paſs through the ſame.

DEMONSTRATION.

Becauſe E C is parallel to A B, it will alſo be parallel to O Q; (theorem XI.) wherefore C is their common vaniſhing point. And, by the preceedent, o q, the image of O Q, will, when produced, paſs through the vaniſhing point, and a b, the image of A B, when produced, alſo paſſes through the ſame; therefore the images a b,

o q, of the two original parallel lines A B, O Q, when produced, meet in that ſame vaniſhing point C. *Q. E. D.*

COROLLARY I.

The ſame vaniſhing point can not belong to any two original lines which are not parallel to each other, for the ſame line, drawn from the point of ſight to meet the picture, can not be parallel to more than one of them.

COROLLARY II.

The center of the picture is the vaniſhing point of all lines which are perpendicular to the picture, becauſe the axis of the eye is parallel to all ſuch lines. (def. V. part I. and theorem VII. part II.)

THEOREM VII.

Original lines, whoſe directing points fall in the ſame director, have parallel images, and the angle made at the point of ſight, by the directors of any two original lines, is equal to that formed on the picture by their images.

SAME FIGURE.

Bring D to coincide with D, raiſe up the plane D Y, and make the picture paſs through both the planes D Y and E M; now let N M and O Q be two original lines, having their directing points D and T each in the ſame director E D; then are the images n m, o q, of thoſe two lines, parallel to each other.

DEMONSTRATION.

For the lines C P, V S, are each parallel to the director E D, whence the images n m, o q, being in thoſe lines, are alſo each parallel to the ſame director E D; they are therefore parallel to each

other; and when there are different directors, as all directors meet in the point of ſight, they will alſo be parallel to the images.

Q. E. D.

THEOREM VIII.

If two original lines meet each other, their images will alſo meet each other in that point of the picture, which is the image of that point in which the originals meet one another.

PLATE IV. Fig. 5.

Form the figure according to the directions annexed to definition XXVI.

DEMONSTRATION.

Let B G, A G, be two original lines meeting in G, then are b g and a g the images thereof. Now, ſeeing G is a point common to both the lines A G, B G, the image g of that point will alſo be common to both the images a g, b g; thoſe images therefore meet in the point, which is the image of that point in which the originals met each other. *Q. E. D.*

COROLLARY.

The images of all parallel lines whatever are either parallel, or meet in ſome one point; for, when the originals are parallel to the picture, their images are alſo parallel, and when not parallel to the picture, their images meet in one common vaniſhing point, as was ſhewn above.

THEOREM IX.

All lines in an original plane have their vaniſhing, interſecting and directing points, in the vaniſhing, interſecting and directing lines of that plane.

PLATE III. Fig. 4.

Let the picture now pass through the plane D Y only, and the plane G X pass through the picture, and let the plane F M be the original plane, and A B, M N, two given lines in that plane.

DEMONSTRATION.

The lines S P, F R, in which the picture and directing plane are cut by the original plane F M, are the only lines which can be cut by lines drawn in the original plane, as M N, A B, &c. wherefore P and S are the intersecting points of those lines, and D the directing point thereof; for these lines meeting, in this case, the directing line in the same point D, have their directing point common. Now E C, E V, being drawn parallel to A B and M N, determine their vanishing points C and V, but these lines are also parallel to the original plane; wherefore the plane G X, drawn through those lines, will also be parallel to the plane containing the lines A B, M N; and as no other plane can be drawn through E parallel thereto, it will be the vanishing plane of the original plane, and the line H L the vanishing line thereof; but C and V, the vanishing points of A B and M N are in that line; therefore the proposition is manifest.

Q. E. D.

COROLLARY I.

The image of any point, line or figure, situated in that part of an original plane, which is beyond the intersecting line, must fall on the picture between the intersecting and the vanishing line, as is evident by what was premised above in cor. II. to theorem V.

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From the above it appears, that when the ground is considered as the original plane, the vanishing line thereof will be the ultimate

extent to which the image of any part of the ground, eſteemed as level, can extend; this line therefore, in that caſe, determines the apparent boundary of the horizon, whence, being taken in this ſenſe, it is called the horizontal line; on the ſame ſuppoſition, the interſecting line is called the ground line, as being then the interſection of the picture with the ground. But if we ſtop here, at this imperfect and confined way of conſidering and applying planes, as adapted by authors of this kind of perſpective, the art will be inſufficient in many caſes, and in general intricate and complex; which a due application of the improvements, furniſhed by the principles here treated of, will render perfect, and make the art of drawing perſpective repreſentations ſimple, eaſy and univerſal.

COROLLARY II.

Any two original lines, in the ſame plane, which are not parallel to the picture, having their images parallel to each other, have a common directing point; for only one director can be drawn parallel to both their images, and that director can cut the directing line of the original plane only in one point; that point is therefore the common directing point of the two original lines. And when any two lines in an original plane, cut the directing line of that plane in the ſame point, their images on the picture will be parallel.

COROLLARY. III.

Lines, drawn through any two vaniſhing points, any two interſecting points, and any two directing points, of lines in an original plane, are the vaniſhing, interſecting and directing lines of that plane; for any two points, in a line being given, the whole line is thereby determined.

COROLLARY IV.

All original lines, parallel to an original plane, have their vaniſhing points in the vaniſhing line of that plane; for lines may be drawn in the original plane parallel to the propoſed lines, and all parallel lines, having the ſame vaniſhing points, (theorem VI. part II.) the vaniſhing points of the propoſed lines are therefore in the vaniſhing line of that original plane to which they are parallel. (theorem IX. part II.)

THEOREM X.

The vaniſhing line of a plane, perpendicular to the picture, paſſes through the center of the picture, which it can not do when that plane is in any other poſition.

DEMONSTRATION.

For the vaniſhing plane will then paſs through the axis of the eye, which is perpendicular to the picture, (def. V. part II.) and all planes, which paſs through the axis of the eye, are perpendicular to the picture, wherefore thoſe planes can not be parallel to planes which are not at right angles to the picture; therefore the vaniſhing planes thereof do not paſs through the axis of the eye, conſequently the vaniſhing lines of ſuch planes do not paſs through the center of the picture. *Q. E. D.*

THEOREM XI.

All parallel original planes have the ſame vaniſhing line, center and axis of the eye, and their interſecting and directing lines are parallel to each other.

PLATE VI. Fig. 7.

Make S, in the plane S L, coincide with S in the plane A K;

make A coincide with A, and the plane G P pafs through the plane S L, in the line R T; raife up the plane K L, fo that the plane G P may pafs through the line U O, and the two lines K P coincide with each other; turn the plane K L about, till K coincides with K, then bring D, in the plane D Y, to coincide with D in the plane A K, and make the plane Y G pafs through the plane S L in the line W X, and pafs the plane F M through the plane K M in the line M V, and pafs the plane V l i through the plane S L in the line H L.

Now, let the plane S L repreſent the picture, K M the directing plane, E the point of ſight, A K one original plane, and Y G another parallel thereto, then are the interſecting and directing lines S I, K P, parallel to each other, and both thoſe planes have the ſame vaniſhing line F N, the ſame center c, and axis of the eye E c.

DEMONSTRATION.

For the plane F M, paſſing through E, the point of ſight, parallel to either of the given planes, will alſo be parallel to the other; it is therefore the vaniſhing plane of both the ſaid given planes. (def. VIII. part II.) Now the ſame vaniſhing plane can produce but one vaniſhing line F N, and E c is the axis of the eye belonging thereto, and c the center thereof, and the interſecting and directing lines S I, K P, being parallel to the ſame vaniſhing line F N; they are therefore parallel to each other. (theorem XI. part I.) *Q. E. D.*

COROLLARY I.

Planes, which are not parallel one to another, can not have the ſame vaniſhing line; for the ſame vaniſhing plane can, in ſuch caſes, be parallel only to one of them.

COROLLARY II.

All parallel original planes have the ſame vertical line, vertical plane, and parallel of the eye; for there can be but one line drawn through the center of the picture, perpendicular to the ſame vaniſhing line, nor but one line through the point of ſight, parallel to the ſaid vaniſhing line; neither can two different vertical planes paſs through the ſame point of ſight and ſame vertical line.

THEOREM XII.

If two original planes cut each other in a line parallel to the picture, their vaniſhing lines will be parallel to each other, as alſo their interſecting and directing lines, if neither of theſe laſt coincide.

SAME FIGURE.

Let the planes G P, Y G, be two original planes interſecting each other in the line B G, parallel to the picture S L, and let the planes M F, V H, paſs through the point of ſight E, parallel to the ſaid original planes, and then will the vaniſhing lines H L, F N, be parallel to each other, as alſo the directing lines K P, Q Y, and interſecting lines W X, T R.

DEMONSTRATION.

Let the plane A B G paſs through the line B G parallel to the picture and directing plane K M, then will the ſections B G, X W, Y Q, made by the plane Y G, with the three planes A B G, S L, K M, be parallel to each other, (theorem XIV. part I.) and for the ſame reaſon the ſections B G, R T, K P, made by the plane G P, interſecting the ſame three planes, will alſo be parallel one to another, conſequently X W, R T, the interſecting lines of the two given original planes, will alſo be parallel, and their directing lines Q Y, K P, will

alſo be parallel. (theorem I. part II.) Now the vaniſhing lines H L, F N, of thoſe planes are parallel to their interſecting lines, by the aforeſaid theorem, and the original planes, not being themſelves parallel, they can not have the ſame vaniſhing line; (cor. I. to theorem VI. part II.) therefore the vaniſhing lines H L, F N, are diſtinct and parallel to each other. *Q. E. D.*

COROLLARY I.

All planes, whoſe vaniſhing lines are parallel, have the ſame vertical plane, vertical line, and parallel of the eye; for C n, which is at right angles to F N, is alſo perpendicular to H L, and M E, being parallel to F N, is alſo parallel to H L.

COROLLARY II.

If the interſection B G of the given planes were in the picture, it would then be their common interſecting line, and if it were in the directing plane, it would be their common directing line; but, in either caſe, their other lines would be parallel one to another.

THEOREM XIII.

If the vaniſhing lines of two original planes be parallel, their common interſection will be parallel to the picture.

SAME FIGURE.

Let the two original planes be the ſame as in the preceedent.

DEMONSTRATION.

Then (by cor. I. to the laſt theorem) the original planes, having the ſame vertical plane perpendicular to them both, it is alſo perpendicular to their common ſection B G; but the vertical plane is

perpendicular to the vaniſhing lines of the original planes; (theorem II. part II.) therefore B G, the common interſection of thoſe planes, is parallel to their vaniſhing lines H L and F N, and conſequently parallel to the picture. *Q. E. D.*

COROLLARY I.

If an original line B G be parallel to the picture, it will be parallel to the vaniſhing, interſecting and directing lines of all original planes which can paſs through the ſaid original line.

For all planes whatſoever, which paſs through the line B G, cut the plane A G in that ſame line; conſequently the vaniſhing, interſecting and directing lines of all ſuch planes are parallel to B G. (16. 11.)

COROLLARY II.

The original of the image of any line in an original plane, parallel to the vaniſhing line of that plane, is parallel to the picture.

For a line in an original plane, parallel to the picture, being parallel to the vaniſhing line of that plane, by the corollary above, its image muſt be parallel to the ſame line; (cor II. theorem IV. part II. and theorem XI. part I.) therefore a line, as B G, parallel to the picture, may be found in the original plane, which will produce the given image; but no two different lines in the ſame plane can produce the ſame image; therefore, if the given image be the image of a line in the original plane, it muſt be the image of B G, a line in that plane parallel to the picture.

THEOREM XIV.

If an original plane, being produced, paſs through the point of ſight, its vaniſhing and interſecting lines will coincide, and its directing line will be the ſame with the parallel of the eye.

ILLUSTRATION. Plate IV. Fig. 5.

Let B A Z R be the original plane, which, being produced to the line P L, paſſes through the eye at E, cutting the directing plane N Y in the line P L.

DEMONSTRATION.

Now, ſeeing that L B is the only plane which can paſs through L P parallel to the original plane B A Z R, it follows that the original and vaniſhing plane coincide, and form one continued plane; therefore L P is both the directing line and the parallel of the eye, belonging to the ſaid original plane, and x y, the interſection of the picture O P, with the original plane produced, is both its interſecting and vaniſhing line. *Q. E. D.*

COROLLARY I.

The image of an original plane, paſſing through the point of ſight, is a line only, and in that line the images of all points, lines and figures, ſituated in the original plane, muſt be formed; wherefore the original plane, in this caſe, hath no depth. For a line, drawn from E, the point of ſight, to any point in the plane B A Z R, will cut the picture O P in ſome one of the points in the line x y; therefore the image of that point muſt be in that line x y.

COROLLARY II.

All original lines, ſituated in the plane B A Z R, have the ſame line P L for their director. For no line in the original plane can cut the directing plane N Y, but only in the line P L.

COROLLARY III.

Any line, drawn in the picture, may be the image of an original

line in a plane paſſing through the eye, and the line given in the picture; or it may be taken as the vaniſhing line of that plane, or as the image of its interſection with all other planes whatſoever which it interſects, as is evident from the above. (cor. I.)

COROLLARY IV.

The original of a line given in the picture can not be determined, unleſs two points in that line be known, one of which at leaſt muſt be an original point, but the other may be the vaniſhing point of the ſaid original line required.

For if the vaniſhing point alone be given, the direction only of the original line is thence determinable, as being parallel to the line producing that vaniſhing point, but the original line itſelf may be any line parallel to that ſaid line, and being in a plane paſſing through the eye and the line given in the picture; but when the direction of the original line and an original point in that line are known, the original line itſelf is then determined, by reaſon there can not be two different lines drawn through a given point parallel to the ſame line.

THEOREM XV.

The original of any figure in the picture, may be any object which is bounded by the ſame pyramid of rays indefinitely produced.

SAME FIGURE.

Let OP be the picture, E the point of ſight, AGBF the original figure, NL the directing plane, and a g b f the image of the original on the picture OP.

DEMONSTRATION.

Now g c may be the image of any original line in the plane GEC,

terminated by the lines E G, E C, indefinitely produced, and b c may be the image of any original line in the plane C E B, bounded by the lines E C, E B; in like manner produced (cor. III.) to the preceedent, it is the ſame with the other ſides of the given figure; therefore a g b f may be the image of any figure whatſoever, bounded by the ſame pyramid of rays E F B G A, indefinitely produced.

Q. E. D.

COROLLARY.

If the image of any plane figure be given in the picture, its original can not be determined, unleſs three points in that figure be known, one of which at leaſt muſt be an original point, but the other two may be vaniſhing points.

For the two given vaniſhing points will determine the vaniſhing line of the plane containing the original figure; (cor. III. to theorem IX. part II.) conſequently the direction of that plane is known, ſeeing it muſt be parallel to the vaniſhing plane which produces that vaniſhing line; (def. VIII. part II.) but, notwithſtanding this, the original plane is yet undetermined, for it may be any plane parallel to that vaniſhing plane; (theorem XI. part II.) whence ſome one point in the original plane is neceſſary to be known, in order to aſcertain that plane; which point being given, the original plane will be thereby determined, for two different planes can not paſs through the ſame point parallel to the ſame vaniſhing plane. Now the original plane, containing the original figure, being thus found, the original of the given image on the picture is truly aſcertained.

THEOREM XVI.

Any line in the picture, parallel to the vaniſhing line of an original plane, if it be the image of an original line, muſt be either the image

of a line parallel to the picture, or of one whose directing point is somewhere in the parallel of the eye belonging to that plane.

DEMONSTRATION.

For, if the original line be parallel to its image, it must also be parallel to the picture; but, if it be not parallel to its image, that image must then be parallel to the director of the original line; that director, being therefore parallel to the proposed vanishing line, (theorem XI. part I.) it must be the parallel of the eye belonging to that vanishing line, (theorem I. part II.) seeing there can not be drawn two different lines through the point of sight parallel to the same vanishing line. *Q. E. D.*

COROLLARY I.

Any line in the picture, parallel to the vertical line of an original plane, if it be the image of an original line, must be either the image of a line parallel to the picture, or of one, whose directing point is in the director of that plane. For, if the original line be not parallel to the picture, its image must be parallel to its director; which director, being therefore parallel to the proposed vertical line, it must be the director belonging to the original plane.

COROLLARY II.

Any two parallel lines in the picture, if they be the images of any two original lines, they must be either the images of lines parallel to each other and to the picture, or of such original lines as have the same director. For, if the original lines be parallel to their images, they must have the same director, by reason there can be but one director drawn parallel to the given images.

THEOREM XVII.

If two original planes cut each other in a line not parallel to the picture, their vaniſhing, interſecting and directing lines will alſo cut each other; and the interſection of the vaniſhing lines will be the vaniſhing point, the interſection of the interſecting lines will be the interſecting point, and the interſection of the directing lines will be the directing point of the common interſection of thoſe planes.

ILLUSTRATION. Plate VII. Fig. 8.

Bring Q, in the plane K C, to coincide with Q in the plane B M, bring D, in the plane N D, to coincide with D in the plane B M, and make T, in the plane D V, to coincide with T in the plane B M, and make the plane L R paſs through both the planes D V, K C, in the lines V T, C Q; bring D, in the plane D W, to coincide with D in the plane B M, and paſs the plane W H through the plane L R, in the line H L.

Now, let the planes K C, B M, be two original planes, cutting each other in the line K Q, which is not parallel to the picture L R, let O be now the point of ſight, Z B the directing plane, W H the vaniſhing plane of B M, and D V that of the plane K C; then are the lines H L, V T, the vaniſhing, T R, C Q, the interſecting, and P B, P N, the directing lines of the ſaid original planes; I ſay, thoſe ſeveral lines will cut each other, and the reſt be as the theorem declares.

DEMONSTRATION.

Becauſe the line K Q is not parallel to the picture, it muſt have a vaniſhing, interſecting and directing point. Now K Q, being a line common to both the original planes, its vaniſhing point V muſt be in both their vaniſhing lines; (theorem IX. part II.) whence H L, the vaniſhing line of the original plane B M, muſt croſs S T,

the vaniſhing line of the other original plane K C, in the point V, ſeeing thoſe lines can not coincide, by reaſon the propoſed planes are not parallel to each other; (cor. I. theorem XI. part II.) therefore V is their common vaniſhing point.

For the ſame reaſons the interſecting lines T R, C Q, and the directing lines P B, P N, or P Z, muſt cut each other, as here at Q and P, thoſe lines being parallel to their reſpective vaniſhing lines; therefore Q is the interſecting, and P the directing point of the ſaid original planes. *Q. E. D.*

COROLLARY I.

Hence it follows, that, if two original planes be both perpendicular to the picture, their vaniſhing lines will make, with each other, an angle equal to the angle of inclination, which the original planes have one to the other; and their common interſection will be in the center of the picture, which is alſo the common center of thoſe ſaid vaniſhing lines.

SAME FIGURE.

Raiſe up the plane D V, and make the plane L R paſs through the ſame in the line V T, bring D, in the plane D W, to coincide with D in the plane B M, and paſs the plane W H through the plane L R, in the line H L; make the line B W, in the plane D V, paſs through and coincide with the line B W in the plane D W, and revolve the plane E X about the line E I, ſo as to paſs through the plane L R, in the line X C. *E is now the point of ſight.*

Now, let the planes B M, D V, be the two original planes, perpendicular to the picture L R, then will the interſecting lines T R, V T, of thoſe planes with the picture, determine their angle of inclination, which is, in this caſe, a right angle, they being here perpendicular to each other, as well as to the picture; and, as the in-

terfecting lines determine that angle, the vaniſhing lines H L, X C, or X Q, muſt alſo do the ſame, (10. 11.) and the line G T, which is the common ſection of the original planes, being alſo perpendicular to the picture, (19. 11.) its vaniſhing point is in C, which is the center of the picture; (def. V. part II.) therefore C is alſo the common center of the vaniſhing lines of the propoſed planes.

COROLLARY II.

Hence it alſo follows, that when two original planes D V, B M, are perpendicular to each other, as well as to the picture, their vaniſhing lines will be at right angles to each other, and the vaniſhing line of either plane will be the vertical line of the other plane; for the vaniſhing lines H L, X Q, are perpendicular to each other, becauſe they make with each other the ſame angle with that of the propoſed planes, by the preceedent corollary; and the vertical line of every plane, paſſing through the center of the picture, is perpendicular to the vaniſhing line of that plane; (theorem II. part II.) wherefore the vaniſhing lines H L, X Q, paſſing through C, muſt be reciprocally the vertical line one of the other.

COROLLARY III.

If two original planes D V, B M, be perpendicular to each other, and only one of them, ſuppoſe B M, perpendicular to the picture, their vaniſhing lines will ſtill be at right angles to each other, and the vaniſhing line H L, of that plane B M, which is perpendicular to the picture, will be the vertical line of the other plane D V, which is not perpendicular to the picture, but the vaniſhing line of the laſt-mentioned plane D V will not be the vertical line of the firſt-mentioned plane B M, but only parallel thereto; for the plane B M, being perpendicular to the picture, all lines, perpendicular to that

plane, are parallel to its vertical line X Q, and alſo to the picture; and ſeeing the planes B M, D V, are perpendicular to each other, a line may be drawn, in the plane B M, perpendicular to the plane D V, (38. 11.) and conſequently parallel to H L, and alſo to the picture L R; that line being therefore parallel to the vaniſhing lines of all planes which paſs through it, (cor. I. theorem XIII. part II.) it muſt be alſo parallel to H L, the vaniſhing line of the plane B M; (theorem XI. part I.) that vaniſhing line is therefore parallel to the vertical line of the plane D V, and conſequently perpendicular to the vaniſhing line thereof. Now the line H L, paſſing through C, the center of the picture, at right angles to the vaniſhing line of the plane D V, is the vertical line of that plane; but, as the plane D V is, by hypotheſis, not perpendicular to the picture, its vaniſhing line can not paſs through C, and conſequently H L can not, in this caſe, be the vertical line of the plane D V, but parallel thereto only.

THEOREM XVIII.

If an original line be given in a plane, parallel to the picture, it will be in the ſame proportion to its image on the picture, as the diſtance between the eye and original plane; is to the diſtance between the eye and the picture.

PLATE IV. Fig. 5.

Let O P be the picture, c its center, E the point of ſight, A B a line given in the plane S T parallel to the picture, E c the axis of the eye produced till it meets the original plane in C; then, as the original line A B is to its image a b, ſo is E C, the diſtance of the eye from the original plane, to E c, the diſtance of the eye from the picture.

DEMONSTRATION.

Becauſe A B is in a plane parallel to the picture, it is itſelf parallel to the picture, and alſo to its image a b; (cor. II. theorem IV. part II.) wherefore the triangles E A B, E a b, are ſimilar; and, for the ſame reaſon, the triangles E C B, E c b, are alſo ſimilar; whence, in the ſimilar triangles E A B, E a b, we have

A B : a b :: E B : E b,

and, in the ſimilar triangles E C B, E c b, it is

E B : E b :: E C : E c, conſequently

A B : a b :: E C : E c. *Q. E. D.*

COROLLARY I.

If the original line A B be any wiſe divided into ſeveral parts, the images of thoſe parts will have the ſame ratio one to the other, as the original parts bear one to another: becauſe each original part is to its image as the diſtance between the point of ſight, and original plane, is to the diſtance of the eye from the picture, which two diſtances remain invariable; and, for the ſame reaſons, the images of the ſides of any figure A B G F, in a plane parallel to the picture, have the ſame proportion to each other, as the correſponding ſides of the original figure have one to the other.

COROLLARY II.

The image of any figure, in a plane parallel to the picture, is ſimilar to its original; becauſe the angles, made by the ſides of the image, are equal, and its ſides proportional to the correſponding angles and ſides of the original figure, by cor. IV. theorem IV. part II. and the preceedent.

COROLLARY. III.

The image of a given line, in a plane parallel to the picture, will be of the ſame length, in whatever point of the directing plane the eye be fixed; becauſe the diſtance between the eye, the picture, and the original plane, undergoes no alteration, let the point of ſight be taken in the directing plane where it will; therefore the proportion of the image to its original remains the ſame.

COROLLARY IV.

If the picture and original plane are both on the ſame ſide of the eye, the greater the diſtance is between the eye and original plane, the nearer does the image of any given line in that plane approach to equality with its original.

ILLUSTRATION. Plate V. Fig. 6.

Raiſe up the planes P Z, D K, and make the planes M N, S Q, paſs through the plane D K; bring R to coincide with R, and make the plane K A paſs through the planes M N, S Q, P Z.

Now let A D be a given line in the original plane P Z, and S Q the picture, a d its image when ſeen by the eye at E; ſuppoſe now the eye to be removed farther from the original plane, as at K, then, drawing the lines D K, A K, the line F G will be the image of A D, when viewed by the eye placed in K; whence, ſeeing the original line A D is to its image ſeen at E, as E L is to E C, by the preceedent theorem, the ſame line A D will be to its image F G, ſeen at K, as K L is to K C; but K E, being greater with reſpect to E C, than it is with reſpect to E L, (8. 5.) K E + E C = K C, will be greater in reſpect of K E + E L = K L, than E C is to E L; and conſequently F G, the image of A D, when ſeen from K, will be greater in proportion to A D, then it will be when ſeen from E.

Now, ſeeing the image of A D is always leſs than its original when thus ſituated with reſpect to the eye and the picture, and, as the image becomes greater in proportion as the diſtance of the eye is increaſed; it muſt therefore approach nearer to an equality with the original line firſt given, in proportion to the remoteneſs of the point of ſight. *Q. E. D.*

SCHOLIUM.

It having been ſhewn, in cor. I. to theorem V. part II. that the directing point of an original line can have no image on the picture, it follows, that all points whatever of an original line, which can be repreſented on the picture, muſt be at ſome diſtance from the ~~original~~ plane. Now the line on the picture, bounded by the image of any point in an original line, and the vaniſhing point of the ſaid line, is called the INDEFINITE IMAGE of that ſaid original line. And the image of any determinate part thereof, bounded by any two points given therein, is the PERSPECTIVE IMAGE of that determinate line ſo bounded. And the remainder of the indefinite image is called the COMPLEMENT OF THE IMAGE. And that part of the original line, contained between its directing point and that point of the original line which is neareſt the directing point, is called the COMPLEMENT OF THE ORIGINAL LINE.

PLATE VIII. Fig. 9.

Raiſe up the plane G E, and make the picture S H paſs through the ſame, make D coincide with D, and the plane M H paſs through the picture S H, in the line H L.

Now, let A G be an original line, C its vaniſhing point, A the neareſt point thereof to its directing point D; then is a C the indefinite image of A G. If G had been the neareſt point of the line re-

preſented on the picture, g C would then have been the indefinite image of that line. If both the points A and G be repreſented on the picture, as here in the points a and g, the line a g is the perſpective image of that determinate original line A G. Now a g being the perſpective image, g C is the complement of that image, and A D is the complement of the original line A G.

THEOREM XIX.

If an original line be divided into any two parts, the rectangle contained between the extremes of the indefinite image thereof, will be to the rectangle contained between the mean part thereof and the whole indefinite image, as that part of the original line, which is neareſt to the directing point, is to the other more diſtant part thereof.

SAME FIGURE.

Let A G be the original line, any how divided in the point K, into the two parts A K, K G, and the indefinite image thereof, a C, into the three parts a k, k g, g C; I ſay, the rectangle under the extremes g C, and a k, is to the rectangle under the mean part k g and the whole indefinite image a C, as A K, the nearer part of the original line A G, is to K G, the other part thereof.

DEMONSTRATION.

Through a, the image of the point A, draw the line T V parallel to the original line A G, cutting E G and E K in the points T and X, then are the triangles E C k, k a X, ſimilar one to the other; wherefore a X : E C :: a k : k g + g C,

and the triangles E C g, g a T, are alſo ſimilar; whence we have

E C : a X + X T :: g C : a k + k g,

and, by multiplying theſe two proportions together, the product will be

$$aX \times EC : EC \times aX + EC \times XT :: ak \times gC : kg \times ak + kg \times kg + gC \times ak + gC \times kg.$$

But, by (1. 2.) $aC \times kg = kg \times ak + kg \times kg + gC \times kg$;

whence $aX \times EC : EC \times aX + EC \times XT :: ak \times gC : gC \times ak + aC \times kg$.

Now, ſubtracting the antecedents from the conſequents, it is

$$aX \times EC : EC \times XT :: ak \times gC : aC \times kg;$$

that is, $aX : XT :: ak \times gC : aC \times kg$,

but, ſeeing that A G and T V are parallel to each other by conſtruction, it will be $aX : XT :: AK : KG$;

therefore $ak \times gC : aC \times kg :: AK : KG$. *Q. E. D.*

SCHOLIUM.

Although the line A G is here ſuppoſed wholly beyond the picture, the propoſition is nevertheleſs true univerſally, wherever the point A be taken in the line A G, ſo long as the line C F is parallel to E D; for any line whatever, that is parallel to C F or E D, cutting the lines E C, E A, E K, and E G produced if neceſſary, will be thereby divided into parts, bearing the ſame ratio to each other, which the correſpondent parts a k, k g, g C, of the line C F, have one to another.

COROLLARY I.

The image a k of A K, the nearer part of the original line A G, is to k g the image of the farthermoſt part K G, as the rectangle under A K the nearer part, and the whole line G D, or line G A, produced to its directing point D, is to the rectangle under K G and D A, the extremes of the whole line G D.

For, through g draw g Q parallel to A G, cutting the lines E A, E K, in the points R and P; then, if a g be conſidered as an original line, C as its directing point, E C as its director, and Q g as its indefinite image, it follows, from the preceedent theorem, that

$$R P \times Q g : P g \times Q R :: a k : k g,$$

and becauſe Q g and A G are parallel,

$$Q g : Q R :: D G : D A,$$

alſo $R P : P g :: A K : K G$;

therefore $R P \times Q g : P g \times Q R :: D G \times A K : D A \times K G$;

conſequently $a k : k g :: D G \times A K : D A \times K G$.

COROLLARY II.

If the parts A K, K G, of the original line, be equal to each other, then a k, the image of the nearer part, will be to k g the image of the remoter part, as the indefinite image a C is to its complement g C.

For, by the preceedent theorem, $a k \times g C : k g \times a C :: A K : K G$.

Now, ſince by hypotheſis, $A K = K G$,

it will be $a k \times g C = k g \times a C$;

whence ariſes this analogy, $a k : k g :: a C : g C$; wherefore the line A C, in this caſe, is divided into harmonical proportion, by the points k and g.

COROLLARY III.

The hypotheſis being the ſame as in the preceedent corollary, it will be

$$a k : k g :: D G : D A.$$

For, in the ſimilar triangles E V T, E C g, we have

$$E V = a C : g C :: V T : E C = V a;$$

but, by the laſt corollary, $a k : k g :: a C . g C$.

Now, ſeeing V T and D G are parallel by hypotheſis,

$$V T : V a :: D G : D A;$$

therefore $a k : k g :: D G : D A$.

COROLLARY IV.

If the images a k, k g be equal, then A K, the nearer part of the original line is to K G, the farther part thereof, as D A, the complement of the original line, is to D G, the whole line G A, when produced to its directing point D.

For, by the above corollary I. $a k : k g :: A K \times D G : K G \times D A$.

Now, if $a k = k g$, then is $A K \times D G = K G \times D A$;

whence ariſes this analogy, $A K : K G :: D A : D G$.

COROLLARY V.

The hypotheſis being the ſame as in the preceedent corollary, it will be $A K : K G :: g C : a C$;

for the triangles E V T, E C g, being ſimilar, it is

$$E C = V a : V T :: g C : a C;$$

but $V a : V T :: D A : D G$.

And, by the preceedent corollary,

$$D A : D G :: A K : K G;$$

conſequently $A K : K G :: g C : a C$.

CONCLUSION.

HAVING thus ſhewn the ſeveral poſitions of the planes, vaniſhing lines, points, &c. neceſſary for conveying to learners a general idea of the nature of perſpective repreſentations, when formed ſtereographically on the plane of the picture; we now proceed to conſider them in another view, ſuch as is required in the practice of drawing the perſpective repreſentations of any given original objects, or for determining the original objects from having their reſpective repreſentations on the picture given; and in this we are to conſider them each as being bounded by its own proper dimenſions, and ſo placed with relation to each other, as to coincide together, and form one continued plane.

PLATE IX. Fig. 10.

Let there now be given the center of the picture, the height of the eye, the diſtance of the eye from the picture, and the poſition of the original plane with reſpect to the picture. Thus, ſuppoſe P M to repreſent the original plane, L M the picture conſidered as being perpendicular thereto, L X the vaniſhing plane, E the point of ſight; then, upon the paper or cloth, &c. intended for the picture which is to contain the required repreſentations, draw at pleaſure the line M N, and parallel thereto, at a diſtance equal to the given height of the eye, draw the line H L, and perpendicular thereto, through the point of ſight E, draw the line E r, interſecting the lines H L, M N, in the points C and r, and through E, parallel to H L, draw the line Z X; then is M N the interſecting line, H L the vaniſhing line, Z X the parallel of the eye, C r the vertical line, E C the diſtance of the eye from the picture, C the center of the picture, and alſo that of the vaniſhing line H L, they both coinciding together in this particular poſition of the picture. This preparation being made,

the ſeveral planes and lines here mentioned are in the proper ſituations for the practice of drawing the repreſentations required to be determined by the help of thoſe ſeveral planes, &c. As to the particular dimenſions proper to be aſſigned to thoſe planes, we ſhall treat thereof in the annotations which are hereto annexed.

PROBLEM I.

To find the indefinite image of an original line, not parallel to the interſecting line, by help of the vaniſhing and interſecting points thereof.

CONSTRUCTION.

Let the line N I be an original line, meeting the interſecting line M N in the point I; from E draw the line E V parallel to N I, cutting the vaniſhing line H L in the point V, draw the line I V, and it will be the indefinite image required.

DEMONSTRATION.

The original and vaniſhing planes being parallel to each other, (def. VIII. part II.) and the line E V parallel to N I, by conſtruction, the point V is the vaniſhing point of the line N I; (def. XIX.) now the interſecting line M N being common both to the plane of the picture and the original plane, I is therefore the interſecting point of the given original line N I; conſequently I V is the indefinite image thereof, by the ſcholium in page 50.

SCHOLIUM.

The line E V muſt be ſo drawn, that the vaniſhing point of the given line may be on the ſame ſide of the vertical line, to which the inclination of the original line tends towards the interſecting line; thus N I, being the original line, its vaniſhing point V falls between

C and H; but if D M be the original line, then its vanishing point V falls between C and L; and if the original line be perpendicular to the intersecting line, as the lines A N, B D, &c. then will the vanishing point fall in C, the center of the picture.

PROBLEM II.

The indefinite image of a line being given; thence to find the direction of the original, by help of the vanishing plane and intersecting point thereof.

CONSTRUCTION.

Let I V be the indefinite image given; produce the same both ways when necessary, so as to meet its vanishing and intersecting points V and I, and from I draw a line, as I N parallel to E V.

DEMONSTRATION.

Then the line I N, being drawn through the intersecting point I, parallel to the line E V, drawn in the vanishing plane through the point of sight, by construction; it is therefore the direction of the original line required. (def. XIX.) *Q. E. D.*

PROBLEM III.

The common vanishing point of parallel lines in the original plane being given; thence to find the vanishing point of all other lines in that plane, which make a given angle with the lines first proposed.

CONSTRUCTION.

Let T O, S U, be two given original lines parallel to each other, V their common vanishing point, and let S T, U O, be two other given lines, making any given angle S T O, or U O T, &c. with the former. From the given vanishing point V draw a line to the point of sight E; then from E, towards that end of the vanishing line to which

the lines tend, whoſe vaniſhing point is ſought; draw the line E y, making with V E an angle V E y, equal to the given angle, and the point y, in which the line ſo drawn cuts the vaniſhing line, is the other vaniſhing point required.

DEMONSTRATION.

By ſcholium, in page 50, the lines o V, u V, are the indefinite images of the given parallel lines T O, S U, and o y, t y, are thoſe of the lines U O, S T, and the angle V E y = T O U, by conſtruction; therefore V and y are the vaniſhing points of thoſe ſeveral lines. (theorem VI. part II.) *Q. E. D.*

COROLLARY I.

If the original lines propoſed, as A B, N D, be parallel to M N, the interſecting line of the original plane, then the vaniſhing point of ſuch lines, as A D, B N, which make a given angle with the propoſed original lines, may be found thus; through E draw E V, to cut the vaniſhing line in V, on the ſame ſide or ſides of C, to which the required originals are ſuppoſed to tend, making the angle X E V, or its equal E V y, equal to A B N, or D A B, the angle propoſed; then will V be the vaniſhing point of all ſuch lines as B N, and v that of all ſuch lines as A D. For the line E V makes the ſame angle with the eye's parallel E X, as the originals, whoſe vaniſhing point is V, make with the interſecting line of the original plane, or any line parallel thereto. The ſame holds good with reſpect to the line E v, and the lines whoſe vaniſhing point is v.

COROLLARY II.

Hence it follows, that if the original lines propoſed incline ſo much to the interſecting line of the original plane, as to cauſe their

vaniſhing point to be out of reach; yet if the angle, which the original lines make with the interſecting line, be known, the vaniſhing point of lines, which make a given angle with the propoſed original lines, may be found in manner as above; for, drawing E V on that ſide of C, to which the original lines are ſuppoſed to tend, making the angle X E V equal to the angle of inclination which the original lines make with the interſecting line; draw another line E v, cutting the vaniſhing line in v, and making the angle V E v equal to the angle propoſed, and v will be the vaniſhing point deſired, for the ſame reaſons as above.

L E M M A.

The director of a line in an original plane makes the ſame angle with the eye's parallel and directing line of that plane, as the image of the given line makes with the vaniſhing and interſecting lines of that plane. Becauſe the director and the image of the original are parallel one to the other. (theorem VII. part II.)

C O R O L L A R Y I.

The director of an original line makes the ſame angle with the eye's director of that plane, which the image of the original line makes with the vertical line of that plane.

Becauſe the eye's director and vertical line are parallel, as is alſo the director of the original line to the image thereof.

C O R O L L A R Y II.

The indefinite images of all lines whatſoever, whoſe directing points are any where in the parallel of the eye, relating to an original plane, are parallel to the vaniſhing line of that plane.

For the images are parallel to their director, which being the pa-

rallel of the eye, in this caſe, they are therefore parallel to the vaniſhing line of the original plane.

COROLLARY III.

The indefinite images of all lines whatever, whoſe directing points fall any where in the eye's director, relating to an original plane, are parallel to the vertical line thereof, whether the propoſed lines be in or out of that plane.

For the director of the eye, being the director of the propoſed lines, it is therefore parallel to the vertical line.

COROLLARY IV.

If any two lines in an original plane cut the directing line of that plane in the ſame point, their images will be parallel.

For the original lines muſt then have the ſame director.

COROLLARY V.

If the images of any two original lines in the ſame plane be parallel, the original lines, if they be not parallel to the picture, muſt have the ſame directing point.

For there can be drawn but one director parallel to both the given images, and that director can cut the directing line of the original plane only in one point, which point is therefore the common directing point of the propoſed original lines.

SCHOLIUM.

It is eaſy to reverſe this problem, that is, from a directing point given, thence to find another directing point, ſuch that the images of all original lines, which have thoſe points for their directing points, may make in the picture an angle equal to any angle pro-

poſed, only by uſing the directing plane and directors in like manner as the vaniſhing plane and the lines drawn to the vaniſhing line from the point of ſight parallel to the given original lines, were uſed in the above, regard being had to the poſition of the directing point ſought, which muſt fall on the contrary ſide of the given directing point to that, whereto the propoſed images are intended to incline, the demonſtration whereof is deducible from the preceedent lemma and its corollaries.

N. B. The angles determined by this problem, when neither of the original lines are ſuppoſed parallel to the interſecting line, are thoſe comprehended between the two vaniſhing points, or the two interſecting points of the images, as V m v, or M m I, or the correſponding angles of the originals, and not the angles which the originals, or their images make ſide ways, as V m M, or v m I.

DEFINITION.

The angles V m v, M m I, or any others in a like poſition with their correſponding originals, are here called INTERNAL ANGLES to diſtinguiſh them from the angles V m M, v m I, which are called external angles.

PROBLEM IV.

A vaniſhing point being given; thence to find two other vaniſhing points, ſuch that all lines, drawn in the picture from thoſe three points, on the ſame ſide of the vaniſhing line, may, by their mutual interſections, form triangles, whoſe originals ſhall be ſimilar to an original triangle given.

PLATE X. Fig. 11.

Let the ſame things be ſuppoſed as before, and let V be the vaniſhing point given.

CONSTRUCTION.

Draw the line E V, and from E take any part thereof, as E W; upon the line E W make a triangle E W Z ſimilar to the original triangle propoſed, having either of its angular points in E; then produce E Z till it cut the vaniſhing line in ſome point, as H, from E draw another line, as E K, parallel to W Z, cutting the ſaid vaniſhing line in K; then are H and K the vaniſhing points required. Let there now be drawn, from the points H, V and K, any lines, as H m, H p, H s; V n, V m, V p; K k, K n, K m; making by their mutual interſections any triangles, a m p, a b k, a k n, a n m, &c. I ſay, the originals of all thoſe ſeveral triangles are ſimilar to the original triangle propoſed.

DEMONSTRATION.

Becauſe the originals of a p, a m, m p, whoſe vaniſhing points are H, V and K, are reſpectively parallel to the lines E H, E V, E K, drawn from the point of ſight to thoſe vaniſhing points, (def. XIX. part II.) and E K is parallel to W Z by conſtruction; therefore the original of m p is alſo parallel to W Z, (theorem XI. part I.) for the ſame reaſons, the originals of the three ſides a m, a p, m p, of the triangle a m p, being reſpectively parallel to the three ſides E W, E Z, W Z, of the triangle E W Z, and their correſponding angles being equal; (theorem XII. part I.) therefore the original of the triangle a m p is ſimilar to the triangle E W Z, but the triangle E W Z is ſimilar to the original triangle given, by conſtruction; whence the propoſition is manifeſt. The ſame may, in like manner, be proved of each of the other triangles a b k, a k n, &c. *Q. E. D.*

SCHOLIUM.

If it happens, in conſtructing the triangle E W Z, ſimilar to the

original triangle given, that the line W Z be parallel to the vaniſhing line H L, then the original triangle will have but two vaniſhing points, from which drawing any two lines ſo as to interſect each other, and theſe being again interſected by any line drawn parallel to the interſecting line M N, a triangle will thereby be formed, whoſe original is ſimilar to the triangle propoſed.

Suppoſe now, in the ſame figure, S T to be parallel to H L or X Y, then a line drawn through E, parallel to S T, will coincide with X Y; whence the ſide of the original triangle, correſponding to S T, can have no vaniſhing point in this caſe; (theorem IV. part II.) it will therefore be parallel to the interſecting line M N, (cor. I. theorem XIII. part II.) to which its image is alſo parallel; (cor. II. theorem IV. part II.) wherefore any two lines H R, K D, being drawn from the vaniſhing points H and K, any where cutting each other, as in m, any lines g f, n p, drawn parallel to the interſecting line M N, or even the interſecting line itſelf, will, by their interſections with the lines H R, K D, form triangles m p n, m g f, D m R, &c. whoſe originals will be ſimilar to the original triangle given.

COROLLARY

Hence, if three vaniſhing points, or two only, in ſuch caſes as the above, of a triangle be given, the ſpecies of the original triangle may be thence determined.

Thus, ſuppoſe H, V and K to be given, draw the lines E H, E V, E K; then, through either two of thoſe adjacent lines, draw a line parallel to the third, and it will determine the ſpecies of the original triangle; for, drawing W Z through E H, and E V, and the line W Z parallel to E K, the triangle E W Z will be ſimilar to the original triangle, from what has been premiſed, or a line W Q, drawn through E V, and E K, parallel to E H, determines E W Q the ſpecies of the

original triangle; for the triangles E W Z, E W Q, are ſo evidently ſimilar, as not to need a demonſtration thereof; or if H and V be the only two vaniſhing points of the original triangle, it is evident, that by drawing S T parallel to X Y, it will determine the ſame by the triangle E S T, which is alſo ſimilar to the triangle E H K.

SCHOLIUM.

In this problem it is limited, that the lines, drawn from the given vaniſhing points of the given original triangle, be all on the ſame ſide of the vaniſhing line, in order that the triangle, formed by the interſections of thoſe lines, may repreſent a triangle ſimilar to the original; for if thoſe interſections fall ſome on one ſide, and ſome on the other, of the vaniſhing line, the original of the triangle ſo formed will be two diſtinct and ſeparate indeterminate figures.

The reverſe of this problem, that is, from a directing point given, to find two other directing points, ſuch that the images of all original lines, drawn from thoſe three directing points, may, by their mutual interſections, form on the picture, triangles ſimilar to a given triangle, may be eaſily performed, by making uſe of the directing plane, and the directors of the original lines, in the ſame manner as was ſhewn, of the vaniſhing plane and the lines drawn from the eye to the vaniſhing points of the given original lines, as was obſerved in the ſcholium page 60, which is ſo evident, that it needs no farther explanation.

PROBLEM V.

To find the image of any given point in an original plane.

This is done by finding the indefinite images of any two lines in

the original plane which pafs through the given point, for the interfection of thofe images is the image of the point required. (theorem VIII. part II.)

EXAMPLE I. PLATE IX. Fig. 10.

For, let z be the point given, D M, N I, two lines in the original plane, interfecting each other in that point, M v, I V, their indefinite images on the picture, interfecting each other in the point m; then is that point m the image or perfpective repreſentation of the point z, which was to be found.

EXAMPLE II.

To find the perfpective repreſentation of B D, a line perpendicular to the interfecting line M N, or bottom of the picture; draw from one of the extremes, as D, a line D M, any how cutting the interfecting line, as here at M; through E, parallel to D M, draw E v, cutting the vaniſhing line in v, the vaniſhing point of D M, through B, the other end of the given line, draw N I, and parallel thereto draw E V, continue B D till it meet the interfecting line as at i; then drawing M v, I V and i C, the intercepted line b d will be the perfpective repreſentation of B D required.

EXAMPLE III.

To find the perfpective repreſentation of A B, a line parallel to the interfecting line or bottom of the picture.

Draw B i, A æ, each perpendicular to M N, and meeting the ſame in the points i and æ; draw æ C, i C, and the other lines as before; then is a b the repreſentation required.

EXAMPLE IV.

To find the repreſentation of a line, as NB, in an oblique poſition with the interſecting line.

Produce N B to I, and draw the ſeveral lines as before directed, then will the line b n be the repreſentation required.

SCHOLIUM.

Now, ſeeing that all plane right lined figures are bounded by lines, which are either perpendicular, parallel or oblique to the interſection line, the examples here given ſuffice for finding the repreſentations of any given rectilinear figures, the picture, original and vaniſhing planes being in the ſituation here deſcribed, we might now proceed to the finding any other repreſentations, or to the finding of the originals themſelves, from having their repreſentations given on the picture, in every ſituation that can be given to the picture, with reſpect to the original plane; but that would be foreign to the deſign of this little tract, which is not intended to teach the art of drawing perſpective repreſentations in its full extent, but only to remove the difficulties which lay in the way of learners at their firſt entrance on this ſtudy, through their not being able readily to conceive, with ſufficient clearneſs, the actual poſitions of the ſeveral planes and lines neceſſary to be duly diſtinguiſhed and applied in ſtudying the rudiments of this art; wherefore, hoping this ſhort Eſſay will be of ſome aſſiſtance to learners in this reſpect, ſhall now endeavour to ſhew them, by a kind of ocular proof, the exact agreement which ſubſiſts between the practical conſtructions which are uſed in this art, and what we have previouſly advanced under a ſtereographical conſideration. To effect which, let us now ſuppoſe the picture to be raiſed at right angles to the original plane, by bringing P to coincide with P, and Q with Q; alſo ſuppoſe the va-

niſhing plane parallel to the original plane, by making Y coincide with Y, and W with W; then, ſuppoſing A B D N, G F K P R, O U S T, to be three original figures given in an original plane, and a b d n, g f k p r, o u ſ t, to be their reſpective repreſentations on the picture, as inveſtigated by the rules of practical perſpective, when thoſe ſeveral planes formed one continued plane: let now the ſilks be drawn through the point of ſight, they repreſenting ſo many rays of light iſſuing from the extremities of the given objects, and all uniting together in the eye or point of ſight; then, by what was delivered in def. I. part II. it appears, that thoſe ſeveral rays will, by their interſections with the picture, mark thereon the ſeveral points requiſite for determining the required repreſentations of the given original objects; and, ſeeing thoſe rays interſect the picture in the very ſame points as were before found by practical conſtruction, it is plain the reſult is the ſame in both; whence the perfect coincidence between the theory and practice of forming perſpective repreſentations appears manifeſt.

And the ſame may, in like manner, be ſhewn, let the poſition of the picture and the original plane be what it will; whether conſidered as parallel to each other, as obliquely ſituated one to the other, as inclining to, or as reclining from each other, under any given angle whatever, &c. which, it is hoped, will evidently appear from what has been delivered, and is contained in the following annotations; wherefore, having thus endeavoured to elucidate the doctrine of planes as applied to the primary operations uſed in the practice of perſpective, which is the propoſed boundary of this attempt, I now refer to Mr. Kirby's treatiſe afore-mentioned, and thoſe maſters whoſe province it is to exhibit the uſes of thoſe principles under the different modifications required in the practical applications thereof,

leaving to another opportunity the conſideration of what may hereafter appear farther neceſſary for rendering this method of explaining the theory more extenſive.

ANNOTATIONS,

OR

GENERAL COMMENT ON PART II.

I. OF MATHEMATICAL PROJECTIONS IN GENERAL.

MAthematical projections are of two kinds, viz. geometrical and ſtereographical; the former whereof ſhould be firſt underſtood, as being uſeful and neceſſary to the latter.

II. OF GEOMETRICAL PROJECTIONS.

Geometrical projections are conſtructed by drawing lines, parallel to each other, from the ſeveral points of the given objects, cutting the plane of projection either perpendicularly or obliquely, under any angle whatever. In this kind of projection, the place of the eye is not conſidered otherwiſe than by ſuppoſing it very remote, or at an infinite diſtance from the plane of projection; whence it can repreſent only two dimenſions at a time, as length and breadth without thickneſs, or length and thickneſs excluſive of breadth, &c. The projected images of objects differ according as they are ſituated with reſpect to the plane of projection: hence ariſe three caſes.

1. When the objects to be projected are in planes parallel to that of projection. 2. When perpendicular thereto. 3. When inclining to the ſame.

In the firſt of theſe caſes, the images are exactly equal and ſimilar to their originals; in the ſecond, they become ſtraight lines; and in the third, are neither equal nor ſimilar.

CASE I. ILLUSTRATION. Plate I. Fig. 1.

Make F coincide with F, and the planes B M, S H, to paſs through each other in the line H L. Now, let A B E D repreſent an object to be projected on a plane S H parallel thereto. Draw A T, D R, B H, E C, parallel to each other, and perpendicular to the plane of projection S H, meeting the ſame in the points T, R, H, C; then, drawing the lines R C, C H, H T, R T, the figure H C R T is the geometrical projection of A B E D, and exactly equal and ſimilar thereto.

DEMONSTRATION.

The planes B D, R H, are parallel to each other by hypotheſis, B H parallel to E C, and both perpendicular to the plane R H by conſtruction; H C is therefore equal and parallel to B E. In like manner it may be proved, that the lines R C, R T, T H, are reſpectively equal and parallel to E D, D A, A B; and all the angles of the given figure being equal to the correſpondent ones in the projection, therefore is R C H T equal and ſimilar to A B E D. Or if the projecting lines are oblique to the plane of projection, thus;

Let A o, D V, B P, E d, be drawn from the ſame points A, D, B, E, any how inclining to the plane R H, and meeting it in the points o, V, P, d, it may be proved as above, that the figure V o P d is alſo equal and ſimilar to A B E D. *Q. E. D.*

COROLLARY

Hence it appears, that the projection will always be the ſame,

whether the plane of projection be before, behind, above, below, nearer to, or farther from the original plane, provided the projecting lines are parallel to each other.

CASE II. SAME FIGURE.

Suppose now E f p q, in the plane B M, to be the object to be projected on the plane R H, perpendicular thereto; then, drawing f r, E C, to meet R H perpendicularly, and they will give the line C r, for the projection of E f p q, for all the lines that can be drawn from any points in that figure, to meet the said plane perpendicularly, will fall in the common section C r; therefore the projection, in this case, is a straight line only.

CASE III. SAME FIGURE.

Make the plane S H pass through the plane D Y; let now the plane D Z be the plane of projection, D Y an original plane containing a line a o, inclining to the plane D Z.

Draw o R, a e, perpendicular to D Z, and they determine the projection e R of the given line a o.

SCHOLIUM.

In this case, when the projecting lines are perpendicular to the plane of projection, each line in the given figure will be, to its image, as radius is to the co-sine of the angle of inclination. For, from the extreme o, draw o b parallel to e R, and it will also be equal thereto; whence o b a, being a right-angled triangle, o b = e R, is the co-sine of the angle of inclination; therefore it is a o : o b = e R :: Radius : the co-sine of the angle of inclination.

OF THE DIFFERENT WAYS OF APPLYING GEOMETRICAL PROJECTIONS.

There are ſeveral ways of applying this kind of projection, in all which the projecting lines are generally conſidered, as being perpendicular to the plane of projection; whence it is diſtinguiſhed by different names, ſuch as Ichnography or plan, Orthography or elevation, Section, Profile, &c.

ICHNOGRAPHY or PLAN, when it deſcribes the perpendicular ſeat or place which objects have on the ground, not regarding their heights above it. Thus the plan of a town, fort, or any building, is the geometrical deſcription of the perpendicular poſitions which their ſeveral parts have with reſpect to the ground, that being conſidered as the plane of projection.

ORTHOGRAPHY or ELEVATION, when it repreſents the plane or ſurface of objects, according to their breadth and heighth above the ground, excluding the depth or ſpace which they occupy thereon. Thus the elevation of a building is the geometrical deſcription of ſome one front or ſide thereof, deſcribed upon the plane of projection, as to the height and breadth thereof. And if the building be ſuppoſed to be cut by a plane paſſing through the ſame, and the ſeveral parts thereof be deſcribed upon that plane, according to the true meaſures which the parts have through which that plane paſſes; this is called a SECTION of that building.

PROFILE, when it is applied to works of fortification, for deſcribing the ſeveral heights thereof, as they would appear upon a plane paſſing through them perpendicular to the ground.

CHART or MAP, when uſed geographically, to deſcribe the form of any part of the earth, and the poſitions of places ſituated upon the ſame, without conſidering the ſphericity thereof, but looking upon the part ſo deſcribed as being a plane.

In all theſe applications, the plane of projection is conceived as parallel to the plane of the original or thing deſcribed; whence the projection ſhould, in ſtrictneſs, be equal and ſimilar to its original, but, in the practical uſes hereof, it is ſimilar only; for it is chiefly uſed to repreſent or deſcribe large things in a ſmaller compaſs than they really poſſeſs, ſo as to be in any aſſigned proportion to the original, at the ſame time preſerving, as much as poſſible, the true form and poſition of the ſeveral parts thereof.

This kind of projection is alſo uſeful to aſtronomical purpoſes, in projecting the ſphere and its ſeveral circles in plano, in which thoſe circles are conſidered as being in planes, whereof ſome are parallel, ſome inclining, and others perpendicular to the plane of projection; and they are accordingly repreſented on the given plane, by circles in the firſt caſe, ellipſes in the ſecond, and by ſtraight lines in the third.

OF STEREOGRAPHICAL PROJECTION.

Stereographical projection is when the projecting lines, drawn from the original object to cut the plane of projection, are not parallel to each other, but converge together, and all meet in one point.

Objects are here conſidered as ſeen from one certain point, called the point of ſight; whence this kind of projection can repreſent at once, all the three dimenſions, length, breadth and thickneſs; that is, as it were, the ſolidity of objects. Here alſo ariſe three caſes or varieties proceeding from the different poſitions which the point of ſight, plane of projection, and the original plane may have with reſpect to each other.

1. When the plane of projection is between the point of ſight and the object.

2. When the object is between that point and the plane of projection.

3. When the ſaid point is between the object and the plane of projection.

The firſt of theſe caſes is called perſpective, and here the image is always leſs, in the ſecond it is always greater, and in the third it may be either leſs, equal, or greater than the given object. But it is not true, that the PARTS OF A MAGNITUDE MAY BE LESS, EQUAL TO, OR GREATER THAN THE MAGNITUDE ITSELF *. It is only the repreſentation of magnitude that is here deſcribed.

ILLUSTRATION. Plate V. Fig. 6.

Raiſe up the plane K D, and make the planes S Q and M N paſs through the ſame; raiſe up the plane P Z, bring R to coincide with R, and paſs the plane K A through the planes M N, S Q and P Z.

Now, in the firſt caſe, let E be the point of ſight, the plane S Q, the picture or plane of projection parallel thereto, and P Z the original plane containing an original line A D. Then, drawing the lines E A, E D, they will interſect the picture or plane S Q, in the points a and d, and thereby determine the line a d for the perſpective appearance of the original A D. Now it is evident, that a d muſt always be leſſer than A D, and that if the plane S Q be ſituated nearer the plane P Z, every thing elſe being as before; the image a d will be greater and greater, as that plane approaches nearer to the plane P Z, and reverts into the original, by the coincidence of thoſe two planes. On the other hand, if the plane S Q be ſuppoſed to approach, in like manner, the plane M N, the image decreaſes, and when thoſe two planes coincide together, it will vaniſh into the point E.

* See page 27. Elements of Mathematics, &c. for the uſe of the Royal Academy at Woolwich, 1757.

In the ſecond caſe, let now the plane S Q be conſidered as the original plane, a d the original object, and the plane P Z to be the picture or plane of projection, and E the point of ſight, as before; then producing E a, E d, till they meet the plane of projection, as here at A and D, the line A D will be the image of a d, and greater than it, as is evident from the conſtruction itſelf, and becomes ſtill greater, as the plane S Q comes nearer to the plane P Z, and will become infinite, or rather be no image at all, when thoſe two planes coincide; for then the lines, which ſhould produce the image, will be parallel to each other, as in geometrical projections, therefore can not meet in a point to form an image, which ſtereographical projection requires them to do.

Laſtly, ſuppoſe B S to be the original object in the plane R W, and S Q the plane of projection parallel thereto, and E the point of ſight; then drawing through E the lines B E, S E, producing them till they meet the plane of projection, as here in the points d and a, and they will determine the line d a for the image of B S. Now, when K E = E C, it is evident that the image d a will be equal to the original B S, as in the poſition before us; but when K E is leſs than E C, the image d a will be greater than B S; but if E K be the greater, then the image will be leſſer than the original, and that when E C is nothing, or that the planes M N and S Q coincide, the image then vaniſhes into the point E; but if E K becomes nothing by the coincidence of K with E, the image ſhould then be infinitely great, and therefore can not be repreſented on the plane of projection.

COROLLARY.

Hence it appears, that, in all theſe three caſes above-mentioned, the point of ſight, plane of projection, and the original plane, muſt be at ſome diſtance from each other; for if either the plane of pro-

jection, or the original plane, coincides with the eye, no image can then be formed; and, if the plane of projection and original plane coincide, the image then changes into the original object.

OF SPHERICAL PROJECTIONS.

In projecting the sphere in plano, the geometrical and stereographical projections are introduced, and applied sometimes singly, and sometimes both together; thus, suppose the eye to be placed in one of the poles or extreme points of the axis of the sphere, and the plane of projection to be perpendicular to that axis, and passing through the opposite pole or other extreme thereof; in this case, the whole sphere will be included between the eye and the plane of projection; and therefore the second case only is here applied.

But the eye continuing in the same position, and the plane of projection supposed to pass through the equator, or any of the lesser circles parallel thereto, as the tropics, polar, or parallels of latitude, &c. then a due application of both the first and second cases is requisite; for the projections of those parts of the sphere, which are between the eye and plane of projection, are derived from case II. and those which are beyond that plane, from case I. Because the eye is here supposed to be in one of the poles of the sphere, it is manifest, from the above, that the said pole can not be represented in the projection, by reason those parts of the sphere, which are nearest the eye, will be projected farthest out, and recede infinitely from the center of the projection, proportionably to their vicinity to the eye; whence these kind of projections are generally confined to some particular circle of the sphere, taken at pleasure, according to the parts which the projection is to describe, for then none of the parts between that circle and the eye are delineated on the plane of projection. Now, if the plane of projection pass through that particular circle,

the whole projection is perſpective, and comes under caſe I. In all theſe inſtances the projections are ſimilar one to another, and differ only as being one greater or leſſer than the other, according as the plane of projection is ſuppoſed to be more or leſs diſtant from the eye.

Having thus deſcribed the nature of mathematical projections in general, I now proceed to conſider caſe I. more particularly, as being the principal deſign of this little tract, referring the ſpherical projections, and what I have prepared for illuſtrating and explaining that ſubject more fully, to another opportunity.

OF THE DIFFERENT APPELLATIONS GIVEN TO PERSPECTIVE.

Authors have conſidered and diſtinguiſhed Perſpective variouſly. Some according to the different poſitions which the picture may have with reſpect to the ground; thence denominating it to be direct, inclining and horizontal, according as the picture be ſuppoſed either perpendicular, oblique, or parallel to the ground. Others from the different poſitions of the picture with reſpect to the eye, as being either directly in, above, below, or on one ſide of the axis thereof. And others, again, by the different diſtances of the eye from the picture. But no advantage has accrued to the art from any of theſe varieties, there being no eſſential difference in the practice, whether it reſult from one or the other of thoſe conſiderations, for the rules ſerve alike in all of them, let what will be the poſition of the eye and the picture, or that of the picture with reſpect to the objects.

For it is only from the different form which the picture may be of, that any eſſential difference can ariſe, as whether it be a plane, concave, or convex ſurface, &c. Such are the caſes of appearances

deſcribed on cupolas, arched cielings, uneven walls, or theatrical ſcenes, &c. which laſt is commonly called Scenography, and conſiſts in making the deſcription upon ſeveral different planes, differently poſited, and at different diſtances with reſpect to the eye. And ſometimes objects are repreſented by reflexion, by ſo drawing them on the picture, that although they then appear confuſed, and without any veſtiges of a regular deſign, yet, by placing a poliſhed piece of metal or glaſs, of a ſuitable form, and in a proper poſition, the ſeveral parts of the picture will be thereby duly reflected to the eye, in their proper ſituations, and the true image of the original, from whence they were drawn, will be perfectly exhibited to view, however confuſed and unintelligible it ſeemed before. For, whether the rays which compoſe an image be reflected, refracted, or any other way diſturbed from proceeding in a rectilinear courſe to the eye, if they be ſo ſubjected by any contrivance, as that they at laſt duly unite in the eye, they will thereby diſcover the true image of that object; but theſe are ſpeculative matters more for curioſity than real uſe, ſhall not therefore digreſs farther on this head, but leave the reader, who is deſirous to learn this particular ſpecies of Perſpective, to the directions of a treatiſe, compoſed by a jeſuit, titled, LE CURIEUSE PERSPECTIVE.[1][2][3]

Some authors have alſo introduced a third kind of projection, called by them MILITARY PERSPECTIVE, or geometrical elevation, by which they pretend to repreſent length, breadth and depth to the view all at the ſame time, by raiſing the ſides of the objects from their ichnography or geometrical plan, adhering to the true meaſures of thoſe ſides, regardleſs of their different diſtances from the eye, but varying the angles of elevation by the laws of ſtereographical projection. The effects of this unfit mixture of both kinds of projection, is an inconſiſtent medley, both unnatural and diſa-

greeable to the eye, and can anſwer no purpoſe but what may be better attained by true Perſpective, is therefore unworthy of a place in this tract; but a more particular account thereof may be ſeen in the firſt volume of the jeſuit's Perſpective, printed at Paris in 1679, to which we refer thoſe who would learn this kind of projection, which we here think not worth our while to explain any farther.

II. OF THE SEVERAL PLANES NECESSARY TO BE CONSIDERED IN DETERMINING THE APPARENT POSITIONS OF OBJECTS ACCORDING TO THEIR RESPECTIVE SITUATIONS.

The fundamental principle, upon which all manner of projections depend, conſiſts in knowing how to determine or mark out, upon any given plane, the place of any point of a viſible object, as ſeen by the eye in any given poſition. Now the apparent place of any point of the object will be in that point of the given plane where a line, drawn from the eye to the ſaid point, interſects the ſaid plane, or when the given plane is beyond the object, in that point where that line duly produced meets the ſaid plane: and the poſitions of any points being obtained, thoſe of lines are eaſily derived, as being terminated by points; ſurfaces, as bounded by lines; alſo ſolids, as being contained under one or more ſurfaces; wherefore the rules of projection have their origin from hence. But the real poſition of a point in abſolute ſpace can not be known otherways than by comparing it with other objects, whoſe ſituations are given; whence ſeveral planes, differently ſituated one to the other, muſt be aſſumed, ſo that, by means of one of them, the poſition of the object may be compared with reſpect to its being even with, above, or below the eye, and by help of another, whether it be in the ſame right line with the eye, or on either ſide thereof. Thus aſtronomers ſup-

poſe a plane, which they call the viſible horizon, to paſs through the eye, and extended every way till it coincide with the heavens; by this they diſtinguiſh the poſitions of the moon, ſtars, &c. either as being in this plane, or as being elevated or depreſſed above or below the ſame; alſo another plane, in like manner extended, they ſuppoſe to paſs through, and at right angles to the former, and thereby diſtinguiſh objects as being to the eaſt or weſt, or on the right or left hand of the obſerver; and many other poſitions relating to the ſun, moon, ſtars, &c. are aſcertained from ſuch like imaginary planes, as is made appear in the aſtronomical ſcience; ſo likewiſe, in Perſpective, three planes are neceſſary for determining the true appearances and poſitions of objects.

ILLUSTRATION. Plate IV. Fig. 5.

Raiſe up the plane D C, and make the plane O P paſs through the ſame, then raiſe up the plane S T; bring D, in the plane P D, to coincide with D in the plane D K, and make the plane L R paſs through both the planes O P, S T; again, bring D, in the plane D Y, to coincide with D, and make the plane M N alſo paſs through the ſaid two planes O P, S T.

Now, ſuppoſe the plane S T to be an original plane, and AGBF to be an object ſeen in that plane by the eye at E, and let L R be a plane paſſing through the eye, and cutting the object, ſo that the line E C, drawn from the eye to the object, be perpendicular to the plane containing that object; let now the plane O P be placed ſomewhere between the object and the eye, and let M F be another plane, interſecting the two other planes at right angles, then will the plane L R ſhew what parts of the object are above, what below, and what are even with or in the plane of the eye; the plane M F ſhews what parts thereof are on the right, and what on the left

hand ſide thereof; and the plane O P ſhews the perſpective appearance or image thereof, when projected by lines drawn through the ſame, from the ſeveral points of the object to the eye, as ſeen in that poſition.

Now, if L B be conſidered as a level or horizontal plane, cutting the picture or plane O P perpendicularly in the line a b or x y, that line is called the horizontal line; and if the plane M F be conſidered as a perpendicular or vertical plane, cutting the ſame in the line H I, this line will be called the vertical line; now the vertical and horizontal planes interſect each other in the eye's axis, or line E C; whereof that part E c, which is intercepted between the eye and the picture O P, is called the principal ray, or diſtance of the eye from the picture. Now, when the three planes O P, L B, M F, are given in poſition, the horizontal and vertical lines are alſo given in poſition on the plane of the picture; wherefore if E, the place of the eye in the interſection of the horizontal and vertical planes, be known, that is, if E c, the diſtance between the eye and the picture be given, and the poſition of any point, ſuppoſe B, be determined from any known diſtances thereof from the three before-mentioned planes, its poſition or place on the picture may from thence be eaſily obtained: thus, draw B C perpendicular to the vertical plane M F, and C G perpendicular to the horizontal plane L B, then will thoſe lines meaſure the diſtances of the point B from the ſaid two planes; then, drawing B E, C E, G E, the figure B C G E will be a triangular pyramid, having the plane of its baſe perpendicular to the vertical plane, and parallel to the ſection b c g, made by the plane of the picture O P; now, ſeeing the line C c expreſſes the diſtance of the ſaid plane of the baſe from the picture, it is alſo the diſtance between the picture and the point B; and, becauſe the two pyramids B C G E, b c g E, are ſimilar,

E C : E c :: G C : g c,
and E C : E c :: B C : b c.

COROLLARY.

Hence it appears, that as the diſtance between the eye and any original object is to the diſtance of the eye from the picture, ſo is the diſtance of the ſaid object, from the horizontal plane, to the diſtance of its image on the picture from the horizontal line. Alſo, as the diſtance between the eye and the original object is to the diſtance between the eye and the picture, ſo is the diſtance of that object, from the vertical plane, to the diſtance of its image on the picture from the vertical line, &c. And if the original plane be between the eye and the picture; thus, ſuppoſe now the plane S T to be the picture, and O P the original plane, then making the difference between E C and E c, the firſt term of the proportions, the reſt will be the ſame as in the two preceeding analogies.

OF THE DIFFERENCE BETWEEN THE TWO WAYS OF CONSIDERING AND APPLYING PLANES, AS USED IN THE OLD AND NEW METHOD OF PERSPECTIVE.

Becauſe the poſition, in which the picture is moſt frequently conſidered by painters, when not concerned with works to be exhibited on cielings, &c. is that of being ſituated perpendicular to the ground or plane of the horizon, it has occaſioned the horizontal plane or ground itſelf to be conſidered as the principal, and oftentimes as the only plane neceſſary to be referred to; thus we ſee the horizontal plane called the geometrical plane, ground, floor or pavement, &c. and the line, in which the picture interſects the ſaid plane, when thus conſidered, called the ground line; now, when the ground and picture are perpendicular to each other, what we call

the vaniſhing line of the ground will then paſs through the center of the picture, (theorem IX. part II.) and the center of the picture will, in this caſe, be the center of that vaniſhing line, (def. XIX.) This vaniſhing line the writers on the old perſpective call the horizontal line, as being the apparent boundary of the viſible horizon conſidered as a level plane, as was before obſerved in the ſcholium to theorem IX. part II. Now, ſeeing the poſitions of objects, whoſe perſpective repreſentations are required, are many of them placed on the ground, and elevated perpendicular thereto, as the front and ſides of buildings, &c. ſuch objects may therefore be conceived, as being in planes perpendicular to the ground; whence the vaniſhing lines of thoſe planes will, in ſuch caſes, be perpendicular to the line thus denominated the horizontal line, and the center of thoſe vaniſhing lines will alſo be in the ſaid horizontal line. (cor. I. theorem XVII. part II.) When the picture, as thus conſidered, is ſo ſituated with reſpect to the original objects, as that they have one line or ſide thereof, as the front of a building, &c. parallel thereto; and the other lines or ſides thereof, as the ſide fronts, &c. perpendicular to the picture; then the vaniſhing lines of thoſe ſide fronts, as well as the horizontal line, will paſs through the center of the picture, which center will be the common center of both thoſe vaniſhing lines, (cor. II. theorem XVII. part II.) and the vaniſhing point of the common interſection which ſuch planes make with the ground, as well as that of all other lines, in any of thoſe planes, parallel to their interſection with the ground, will alſo be in the center of the picture (cor. I. of the ſame); whence it appears, that the center of the picture is of conſiderable uſe in determining repreſentations on the picture, when thus ſituated with reſpect to the original plane, and is therefore called the principal point, or point of ſight; but, in the method we eſpouſe and wiſh to recommend,

the point of ſight is that which is deſcribed in def. IV. part II. and the axis of the eye being the line which produces the vaniſhing point of all lines perpendicular to the picture, it is therefore the line which produces the vaniſhing point of all thoſe lines which moſt frequently occur in this ſituation of the picture; whence it is called the principal ray, and the length of this line being ſet off from the principal point, both ways upon the horizontal line, will give two points, called the points of diſtance. Now, in this particular ſituation of the picture, with reſpect to the original objects, the principal rays being either perpendicular or parallel to the ground or to the picture; they will therefore make the ſame angle with the ground line, as the original lines themſelves make with the ground, (cor. IV. theorem IV. part II.) and thoſe lines which are parallel to the ground, but not to the picture, will have their vaniſhing points in the horizontal line. (cor. IV. theorem IX. part II.) Now, when theſe points fall beſide the principal point, they call them accidental points in the horizontal line; and when it is required to find the repreſentations of lines, which are neither parallel nor perpendicular to the picture, nor to the ground, they generally ſeek their images, by having recourſe to the ſeats which ſuch lines have on the ground, for thereto they moſt commonly refer all ſuch lines as do not come within the reach of the above deſcription, without enquiring after their vaniſhing points, or the lines which ſhould produce them. But theſe principles are too confined for general uſe, and can ſerve only in this particular circumſtance, of conſidering the picture as being perpendicular to the ground; whereas the practice of this art requires that any other plane, and not the ground only, may be taken as an original plane, and the picture may have other ſituations with reſpect to the ground, or the original objects, ſuch as being parallel thereto, inclining to,

or reclining from the ſame, &c. for the parts of a building, as roofs, &c. are not always either parallel or perpendicular thereto; and there are alſo caſes which render it neceſſary to conſider, in the ſame picture, ſeveral different original planes, each having its own interſecting, vaniſhing, and the ſeveral other lines and points peculiar thereto, by means whereof many operations are eaſily performed, which would otherwiſe be extremely tedious and difficult, if not, in ſome caſes, impracticable; but, that we may candidly favour this old ſyſtem of Perſpective, as far as propriety will permit, will allow the horizontal line and ground line to be proper terms for expreſſing the vaniſhing and interſecting lines of the ground, when that is taken for the original plane, but think it abſurd to give theſe appellations to any other plane than the ground; for the original plane may be ſuch, that the vaniſhing line thereof ſhall interſect the picture either in, above or below the horizontal line, or it may be that it croſs or interſect the ſame, as may be thus exemplified.

PLATE VI. Fig. 7.

Bring A to coincide with A, make the planes G P, S L, paſs through each other in the line T R; make K, in the plane K L, coincide with K in the plane K A or K B, paſs the plane F M through the plane K L, in the line V M, and paſs the plane M H through the plane S L, in the line H L. Now, let the plane G P repreſent an original plane, inclining to the picture S L; let the plane F M be conſidered as an horizontal plane, paſſing through the eye at E, and interſecting the picture in the line F N, then is the line F N the horizontal line: let now a plane, as M H, paſs through the eye, and be parallel to the ſaid original plane G P, cutting the picture in the line H L, then is the plane M H the vaniſhing plane of the original G P, (def. VIII.

part II.) and H L the vaniſhing line of the ſame, (def. XI. part II.) conſequently the vaniſhing line falls above the horizontal line in this caſe; and if the original plane had declined from the picture the contrary way, the vaniſhing line thereof would then have cut the picture in a line below the horizontal line, as is eaſy to conceive from what has been here ſhewn, and when the original plane meets the picture at right angles, the horizontal and vaniſhing lines thereof coincide, or fall together in one and the ſame line only; ſuppoſing, as is here done, that one ſide of the original be parallel to the picture, for, if all the ſides thereof are oblique thereto, its vaniſhing and horizontal lines will then interſect each other, as was ſhewn in theorem XVII. part II. the vaniſhing line H L in that figure being now the horizontal line.

The ſeveral efforts, excluſive of the laſt, which have been made in oppoſition to the endeavours uſed for conveying and eſtabliſhing a general knowledge and uſe of this new method of Perſpective will, I hope, ſufficiently juſtify me in this attempt, to ſhew the truth and univerſality thereof; and thus endeavouring to ſet learners right in their firſt notions of the different methods here alluded to, by warning them of the defects under which this art will labour, when diſrobed of the improvements added to it, by conſidering the poſitions of planes in a general view, and thence making the neceſſary practical applications ſuitable for obtaining the advantages it offers to us.

OF THE POSITION OF THE PICTURE WITH RESPECT TO THE ORIGINAL OBJECTS.

Although the picture may have any poſition with reſpect to the objects which are to be repreſented thereon; yet, in order that it may ſhew an agreeable and natural repreſentation of the things deſcribed, it ſhould be ſo placed, that the ſeveral objects may appear

in their natural ſituation, that is, ſuch objects as would really be ſeen by rays parallel to the horizon, or by rays inclining upwards above the ſame, or by others declining downwards below it, may appear on the picture by rays in ſimilar correſpondent poſitions; for the ground or plane of the horizon being the natural or apparent ſeat of all viſible objects, whether terreſtrial or celeſtial, the mind judging them to be higher or lower, according as they are more or leſs elevated above it; the ſituation of all objects may and ought therefore to be referred to that plane, and have ſuch poſitions given them in reſpect thereto, as is agreeable and conſiſtent with their natural appearance; wherefore the moſt proper diſtinctions, to be made of the different ſituations which may be given to the picture, ariſe from its poſition with reſpect to the ground or plane of the horizon, and are therefore three, perpendicular, parallel, or inclining.

OF THE PERPENDICULAR POSITION.

The perpendicular ſituation of the picture is beſt adapted for repreſenting the ground itſelf, and the ſeveral objects which are erected thereupon; and this being a poſition which is parallel to the uſual poſture of the body of the ſpectator wherein the eye is moſt uſed to view original objects, it follows, that the repreſentations hereby formed on the picture, appear the moſt natural and agreeable to the originals themſelves. In this poſition the ichnography of the deſign on the ground is conſidered as deſcribed on a plane perpendicular to the picture, the vaniſhing line whereof paſſes through the center of the picture, (theorem X. part II.) and is parallel to the horizon, and alſo repreſents the ſame, is therefore, in this caſe, called the horizontal line; the elevations of upright objects are conſidered as being in planes perpendicular to the ground, which ſaid planes may be either perpendicular, parallel or oblique to the picture, but their

vaniſhing lines are always perpendicular to the horizontal line, (cor. II. theorem XVII. part II.) and all lines, which meaſure the perpendicular altitudes of objects, thus ſituated above the ground, are parallel to the picture; but it muſt be here obſerved, that the ground, deſcribed in the picture, is ſuppoſed to be level or truly horizontal; for, if the ground have aſcent or deſcent, the picture continuing perpendicular to the horizon, the vaniſhing line of the ground will not, in either of thoſe circumſtances, coincide with the horizontal line, but will interſect the picture either above or below the ſame, according as the ground is elevated or depreſſed; but, nevertheleſs, the lines, which meaſure the perpendicular heights of the original objects, will yet be parallel to the picture, as being perpendicular to the horizon, and they, with reſpect to the ground, will repreſent the oblique ſupports of the ſeveral points above that inclining plane.

The poſition of the picture here deſcribed is proper to landſcapes, views, buildings, hiſtorical pieces, and in general to all pictures where the ſpectator is ſuppoſed to ſtand on the ground, and having the axis of the eye directed parallel thereto.

OF THE PARALLEL POSITION.

Here are two caſes. 1. When the eye is between the ground and the picture. 2. When the picture is between the eye and the ground.

In the firſt, the axis of the eye is ſuppoſed to be turned perpendicularly upwards, and conſequently the ground, being behind the eye with reſpect to the picture, can not have any part thereof repreſented on the picture; whence all kinds of terreſtrial objects are here excluded, excepting ſuch as may be ſuppoſed to aſcend above the plane of the picture, ſuch as the higher parts of mountains or

buildings, or such other objects as may be conceived in the air, but all their uppermost surfaces must necessarily be concealed from the eye.

Pictures or paintings on flat cielings are of this kind, and those on cupolas or arched roofs may be reduced hereto, they all agreeing with respect to the particular kind of objects proper to be described thereon. A neglect in giving due attention to this particular circumstance, has rendered several performances of this sort disagreeable and unnatural; but a judicious painter will forbear to represent on a picture, in this situation, the sea, ground, part of the floor of a building, or the upper faces of steps, &c. notwithstanding examples of such inconsistencies are to be met with.

In the second case, the eye is supposed to be at some height above the picture, and its axis turned perpendicularly downwards; in this circumstance no part of the sky can appear, nor any other thing but what can be supposed to lie either upon the ground, or between it and the picture, such as the pavement or floor of a building, the plan of a garden, &c. and those parts of the building, or such other things which, standing upon the ground, do not reach up to the picture; whence this situation of the picture is the most confined of any, and seldom used but out of curiosity, as on the floor or pavement of a church or dome, which, by an artful disposition of materials differently coloured, may be made to represent objects proper to that situation, or even the reflected image of the building itself, as appearing in a looking glass, or stagnant water, to an eye viewing it from some convenient place, as a gallery, &c. at the top or upper part of the building.

In both the cases here mentioned, the ichnography of the objects on the ground is described as on a plane parallel to the picture, and is therefore similar to its original; the planes of the elevations, and

the lines, which meaſure the perpendicular altitudes of the objects above the ground, are perpendicular to the picture; wherefore the vaniſhing lines of thoſe planes paſs through the center of the picture, which center is the vaniſhing point of all the ſaid lines of altitude; now, ſeeing the ground can not interſect the picture in either of thoſe two caſes, it is not proper to be uſed as the principal original plane, there being no horizontal line in thoſe caſes, but any of the upright ſides of the building deſigned to be repreſented, or any other ſubſtituted plane perpendicular to the picture, may be uſed for that purpoſe, and the objects required to be repreſented may be thereto referred.

OF THE INCLINING POSITION.

The various inclinations, under which the picture may be placed with reſpect to the plane of the horizon, are innumerable, which will have correſponding effects on the place of the horizontal line and the vaniſhing point of the perpendiculars to that plane; as to the kind of objects proper to be repreſented on a picture of this ſort, they are to be determined from what has been delivered concerning the two poſitions before mentioned, and will differ according as the inclination of the propoſed picture approaches nearer to one or to the other of them; but theſe oblique poſitions are ſeldom practiſed, except neceſſity obliges it, from the particular poſition of the wall or place in which the picture is to be painted or placed.

SCHOLIUM.

Let the poſition of the picture be what it will, if it be ſuch that the horizontal line can appear therein, it ought, in ſtrictneſs, to be ſo placed as that the eye, when in the true point of ſight, may be on a level with that line; for then all the repreſentations, formed

on the picture, will appear as being in their true and natural positions. This may, however, be dispensed with, in some measure; for if a picture be drawn, by supposing it perpendicular to the horizon, and it should be necessary to place it so high, that the axis of the eye can not reach up to a level with the horizontal line, then the true appearance which the picture ought to exhibit may be preserved, by inclining the picture forwards, so that a line may be drawn or conceived to issue from the eye perpendicularly thereto, and meeting the same in the center; for, although the ground represented in the picture does not then appear parallel to the horizon, but as rising upwards, yet the spectator, in viewing and considering the representations according to the relations which they have one to the other, without referring to their true positions, or his own real posture, with respect to other objects which are out of the picture, will readily pass over the defect of this deviation from the true position, so long as the whole picture is every where consistent with itself; and the picture being in this situation will, in some sort, exhibit a resemblance of the appearances which the like objects would shew, if reflected by a well-polished plane mirror, or common looking-glass, placed inclining to the horizon, which makes the reflected ground appear as being elevated or depressed with respect to the real ground; but the reflections of all other objects preserve the same relations with respect to the reflected ground, which the real objects have to the ground itself: for, let the spectator but only imagine that he stands perpendicular to this reflected ground plane, and then all will appear as having a natural situation with respect to the ground on which they are supposed to be placed.

But let it be observed, that this liberty be not used too freely; it is allowable only to detached pieces, which are to be placed where conveniency suits; for, such as are expressly designed for a certain

fixed ſituation, as a wall, cieling, &c. do not admit of this variation from the true poſition, by reaſon ſuch pieces have generally a more immediate relation to the reſt of the building itſelf. Suppoſe, for example, that a picture, deſigned for a poſition which is perpendicular to the horizon, were placed in one that is parallel thereto, as a cieling, &c. the ſpectator will not then be able to conceive it as exhibiting repreſentations of objects according to their natural appearance, without imagining himſelf to be ſtanding erect on a plane which appears perpendicular to the horizon, a conception which requires ſuch a force of imagination as is not eaſily acquired, nevertheleſs, we have ſome examples which render this extraordinary effort of the mind abſolutely neceſſary, before the picture can poſſibly have its due effect.

OF THE DISTANCE OF THE EYE FROM THE PICTURE.

From what has been advanced on the nature of ſtereographical projections, it is manifeſt that the repreſentations, formed on the picture, can not appear to the ſpectator exactly as they ought to do, if the eye be not in the true point of ſight; whence it is evident that they ſhould, in ſtrictneſs, be always viewed from that point; therefore a true determination of the poſition of the eye is ſuch an eſſential requiſite in this art, that no examples or authority whatever ſhould induce us to diſregard it, or be too careleſs about the choice of it.

In a room or place that is bounded on all ſides, whatever be the poſition of the picture, ſuch a diſtance muſt be taken for the eye as is within reach; thus, ſuppoſe the picture were to be on the cieling, the diſtance is then determined by the height of the room, deducting therefrom the height of the ſpectator's eye from the floor; but,

when the place is not thus circumſcribed, the artiſt is more at liberty to make ſuch a choice for this diſtance as ſhall be moſt ſuitable to the grand deſign of making the objects appear to the beſt advantage, and in their natural ſituations. Now, when the picture is in a perpendicular poſition, the artiſt has generally the greateſt room for exerciſing himſelf in making his choice of a proper diſtance to work from; let us therefore briefly enquire into the effects which are produced from different diſtances of a picture in this poſition, for, what we ſhall learn from hence, may be eaſily applied to any other poſition of the picture whatever. In this enquiry I ſuppoſe the height of the eye to be given or taken at pleaſure, and remaining conſtantly the ſame. The eye thus abſtractedly conſidered, being placed at different diſtances from the picture, will produce correſpondent changes in the ichnography and orthography of the original objects; for the ichnography thereof will be affected either with reſpect to the quantity of the whole ſpace which it contains, or the proportions of that ſpace taken up by its different parts, according as they are nearer to or farther from the picture, and alſo as to the apparent breadths of thoſe parts. Now here are two things which ought to be carefully guarded againſt: firſt, that the apparent decreaſe of equal parts in the ichnography may not be too ſudden, as they become more remote. Secondly, that ſuch remote parts as are to be diſtinctly expreſſed, may not approach too near the vaniſhing or horizontal line; for if they do, their images will be too ſmall, and too much crouded together, both which contribute to render them indiſtinct, and the picture will be thereby defective; which imperfections are avoided by making a right choice for the place of the eye, or the diſtance thereof from the picture.

LEMMA I.

The distance between the image of any point given in an original plane, and the vanishing line of that plane, is to the vertical line thereof, or to the director, as the line which produces the vanishing point of a line drawn in the original plane through the said given point, is to the distance between the said point and the directing line.

ILLUSTRATION. Plate XI. Fig. 13.

Bring D, in the plane D B, to coincide with D in the plane N B; make S, in the plane S H, coincide with S in the plane N B; make the plane Q B pass through the plane S H in the line F G, and pass the plane Q Z through the plane S H, in the line line H L.

Let A be the point given in the original plane N B; through the same draw the line A B parallel to the interfecting line S I; let Q Z be the vanishing plane of the original plane N B; from the directing point D to any point, as K, in the line A B, draw the line D K, and parallel thereto, from the point of sight, draw the line E C; then is C the vanishing point of the line D K, and E C its distance, P C the vertical line, E D the director, M N the directing line, &c. draw also the lines E A, E B, E K: now A B, being parallel to S I, its image a b is also parallel to S I; and K being a point in the line A B, and k the image thereof, the line a b is that which must contain the image of A, as has been before shewn; but the distance of a, from the vanishing line H L, being equal to k C, and the distance of A from M N is equal to D K, because E C and P C remain constantly the same, it follows, that in what point soever of A B the point A be taken, the distance between the image of that point and the vanishing line will be to P C as E C is to the distance between the given point and the directing line.

COROLLARY I.

The diſtance between the image of any point in an original plane and the vaniſhing line remains the ſame wherever the point of ſight be taken in the parallel of the eye; for A B, a b and X Q being all in the ſame plane, a line drawn from any point in X Q to any point in A B, muſt cut the picture ſomewhere in the line F G, and all points in that line are equally diſtant from the vaniſhing line H L.

COROLLARY II.

If the height of the eye be increaſed or diminiſhed, the eye being in the ſame directing plane, the diſtance between the image of an original point and the vaniſhing line will be increaſed or diminiſhed in the ſame proportion.

COROLLARY III.

If the height of the eye remain conſtantly the ſame, and the diſtance thereof be varied, the diſtance of the image from the vaniſhing line will vary accordingly.

LEMMA II.

The difference between the image of a nearer part of an original line and the image of the part next beyond it, is greater than the difference between this laſt image and that of the next ſucceeding part, and ſo on of others.

PLATE VIII. Fig. 9.

For, let O be the point of ſight, the parts A K, K G, G Y, being equal, q n : n m :: D G : D A, by cor. III. theorem XIX.

and for the ſame reaſon n m : m y :: D Y : D K,

alſo q n — n m : n m :: D G — D A = A G : D A,

n m — m y : m y :: D Y — D K = K Y or A G : D K.

But A G is greater in proportion to D A than it is to D K, therefore q n — n m is alſo greater in proportion to n m than n m — m y is to m y; and n m being greater than m y, therefore is q n — n m ſo much the more greater than n m — m y.

COROLLARY I.

The more remote that the equal parts of the original line are from the directing point, the nearer do the images of thoſe parts approach to an equality, by reaſon their differences are accordingly diminiſhed.

COROLLARY II.

The greater the diſtance of the eye, the more nearly equal do the images become of any two adjacent parts of the original line.

For, ſuppoſe the eye removed from O to E, then, becauſe A K and K G are equal, the image of A K is to that of K G from the point of ſight O, as N G to N A, and, from the point of ſight E, they are as D G to D A; but D G is leſs in compariſon to D A, than N G is to N A; therefore the image of A K is leſs in proportion to that of K G, from the point E, than it is from the point O: but a k, k g, the images of A K and K G, are in the ſame ratio one to the other wherever the eye be placed in the ſame directing plane; therefore, whether the eye be placed at E, or any where elſe, in a plane paſſing through E parallel to the picture, the images of A K, K G, ſeen from thence, will be nearer equal to each other than they would be if formed by viewing them from O.

COROLLARY III.

If, from the interſecting point F of any line F Y, ſeveral diſtances, F A, A K, K G, G Y, &c. be taken, each equal to N F, the

point N being the directing point, q C, n C, m C, y C, the images of thoſe parts, will be in proportion to O N, or F C, as the following fractions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c. and the images of the parts themſelves will be as $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, $\frac{1}{30}$, &c. of F C, the differences of the denominators of each of theſe laſt fractions increaſing from o in this ſeries 2, 4, 6, 8, 10, &c.

In the ſimilar triangles O N A, q F A,
it is N A : F A :: O N = F C : F q,
but N A = 2 F A by hypotheſis; whence F C = 2 F q,
conſequently F q = q C = $\frac{1}{2}$ F C.

Again, in the ſimilar triangles O N K, n F K, we have
N K : F K :: O N = F C : n F,
but F K = $\frac{2}{3}$ N K; whence n F = $\frac{2}{3}$ F C,
conſequently n C = $\frac{1}{3}$ F C,

and in the ſame manner it may be proved, that m C is $\frac{1}{4}$ F C, and y C = $\frac{1}{5}$ F C.

Now, if we call F C = 1, q C = $\frac{1}{2}$, n C = $\frac{1}{3}$, m C = $\frac{1}{4}$, y C = $\frac{1}{5}$, the image n q, which is the difference between q C and n C, will be $\frac{1}{6}$, n m = $\frac{1}{12}$, m y = $\frac{1}{20}$, &c. the difference of the denominators of which fractions increaſe from o in the ſeries 2, 4, 6, 8, &c.

Suppoſe now the parallelogram S Y to repreſent part of the ichnography of the original deſign, ſubdivided into ſmaller ones as in the figure, N F being choſen for the ſpectator's line of ſtation, which determines the poſition of the ichnography with reſpect to the picture; take any height at pleaſure, as N O, for the height of the eye, ſuppoſed now in O, and C O its diſtance from the picture; then, if the images q, n, m, y, of the points or diviſions A, K, G, Y, and conſequently the diſtances of the images of the croſs diviſions of the ichnography, paſſing through thoſe points, ſhall appear too unequal, or fall off too ſuddenly; or that y, the image of Y, the moſt remote

diviſion, appears too far up in the picture, or too near the vaniſhing point C, the fault may be removed by inlarging the diſtance of the eye, and fixing it in ſome other point of the line C O produced, ſuppoſe at E, &c. Now, when C O, the diſtance of the eye, is taken equal to F A, the diſtance of the firſt diviſion from the picture, the heights of the images q, n, m, y, above the interſecting line, will be $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c. of F C, the depth of the original plane, their reſpective diſtances from the vaniſhing line being $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. of that depth, and the images of the parts F A, A K, K G, G Y, &c. will be $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, &c. of that ſame depth; but if, at this diſtance of the eye, the parts near the picture occupy thereon a greater ſpace than is agreeable with reſpect to what is poſſeſſed by the other more diſtant parts, or that the objects, ſituated between G and Y, appear too ſmall and crouded, the diſtance of the eye may be taken equal to the two firſt parts F A and A K together, which will bring the images of the diviſions, next ſucceeding thoſe here mentioned, down to the ſame points on the picture which they themſelves were in before; and thus may there be found ſuch a diſtance for the eye, as ſhall make any point in the ichnography to appear at any propoſed height on the picture, within the limits of the depth of the original plane; but, if it were required to repreſent any particular part of the ichnography, ſuppoſe that between G and Y, ſuch as ſhall occupy the greateſt ſpace in depth poſſible, the diſtance of the eye muſt then be a mean proportional between the two parts F A and F Y; for, at that diſtance, the ſpace A Y will occupy a larger field in depth than it would do at any other diſtance of the eye in the line C O, produced either nearer to or farther from the picture. Hence it appears, that the diſtance of the eye is that which principally commands the diſtance of the moſt remote ground that can be deſcribed with any tolerable diſtinctneſs; for, if the ground be-

yond the picture be divided into ſpaces equal to the diſtance of the eye, the ſeats of all ſuch objects as are contained in the ninth ſpace from the picture, can then occupy no more than the one-ninetieth part of the depth of the original plane, and the image of the extremity of that ſpace being but one-tenth part of that depth diſtant from the vaniſhing line, that one-tenth part is therefore the whole ſpace that is left wherein all ſpaces beyond the ninth can poſſibly be repreſented on the picture, as appears from what has been premiſed, for there being now nine diviſions, the firſt fractions are

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}$;

the ſecond $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}$;

and $9 \times 10 = 90$.

EXAMPLE.

Suppoſe the height of the eye to be 5 feet, and its diſtance from the picture 20 feet.

Then the ſeats of all ſuch objects as are between 160 and 180 feet diſtance from the picture, can poſſeſs no more ſpace than one-ninetieth part of 5 feet, or two-thirds of an inch, and this reaching within 6 inches of the vaniſhing line, or, which is the ſame, the tenth part of 5 feet, it follows, that the ſeats of all objects on the ground, from 180 feet to any aſſignable diſtance, muſt be confined within that 6 inches, ſo that, even at ſo great a diſtance of the eye as 20 feet, the ſeats of objects, 20 feet in depth at the diſtance of 60 or 70 yards, can be repreſented but imperfectly, and all beyond that diſtance will be faint and confuſed; and, if the diſtance of the eye be leſſened, the ſpace which can be diſtinctly deſcribed will be proportionably decreaſed. There is alſo another effect which the diſtance of the eye produces upon the ichnography; for, as by enlarging the diſtance the images of the ſeveral croſs diviſions are

brought lower down towards the interfecting line, ſo their apparent meaſures are proportionably increaſed, and conſequently the lines, which meaſure the breadths of objects parallel to the picture, appear longer, at the ſame time that thoſe, which meaſure their depths, become ſhorter; and, as the apparent breadths of objects are thus encreaſed, ſo alſo are their apparent heights or elevations, thoſe heights being ſuppoſed parallel to the picture; ſo that, upon the whole, ſuppoſing the picture and original objects as retaining their poſition, if the diſtance of the eye be enlarged, their apparent breadths and heights, or thoſe ſides thereof which are parallel to the picture, will be thereby encreaſed, but their depths will be diminiſhed. Now, although we may, from what has been ſaid, eaſily find ſuch a diſtance for the eye as is ſuitable for repreſenting the objects in the moſt advantageous ſituation on the picture, ſo as that the remoteſt parts may not appear advanced too near the horizon, nor their depths decreaſe too ſuddenly, but that a due and agreeable proportion between their apparent heights, breadths and depths may be preſerved in the repreſentations formed on the picture, yet, let it be obſerved, that what has been advanced on this ſubject, is to be underſtood as relating to a picture, formed from a ſcale, equal to the true meaſures of the original objects, that is, when the objects adjoining the picture are repreſented big as the life; but, as this ſcale may be contracted to any dimenſions at pleaſure, it follows, that a picture may be rendered capable of repreſenting much greater diſtances than the above-mentioned, and yet the diſtance of the true point of ſight may be greatly leſſened, thus,

PLATE XI. Fig. 13.

Make D, in the plane D K Q, coincide with D in the plane N B, and paſs the plane S H through the ſame; make B, in the plane

B R, coincide with B in the plane N B; then make the plane Q Z paſs through both the planes S H and B R.

Now let A B M N repreſent the ground plane on which the ichnography of the intended objects is ſuppoſed to be formed in its true dimenſions, and let A B be the interſection of the ground with the ſuppoſed picture A B Z R, and conſequently the neareſt part of the ground propoſed to be repreſented on the picture; now, if D K or E O be the diſtance of the eye neceſſary for rendering the repreſentations ſufficiently diſtinct, and this be too great for the place where the picture is to be fixed, it muſt be drawn from a leſſer ſcale, which may be thus determined.

Between the directing point D and the interſecting line A B draw in the original plane a line, as S I, parallel to A B, on that line erect a plane parallel to the plane or picture A B Z R, and upon the plane thus erected find the perſpective repreſentation of the given plane, the eye being at E, which is here expreſſed by a b z r; then, if a b z r be taken for the picture, E C for the diſtance, and E d for the height of the eye, and that the true meaſures of the objects to be repreſented be all reduced to the ſame proportion as a b bears to A B, then all objects, deſcribed in the picture a b z r, with thoſe proportional meaſures with the laſt-mentioned diſtance and height of the eye, will be exactly ſimilar to a picture of the ſame objects, drawn on the plane A B Z R, according to their true or real dimenſions, having E O and E D for the diſtance and height of the eye, but as much leſſer as is the proportion aſſigned between a b and A B. Thus may the ſcale, and conſequently the diſtance of the eye, be reduced to any convenient meaſure to ſuit the ſituation in which the picture is to be placed.

The ſpace between a b and A B is entirely hid in this caſe, no part thereof being to have a place on the picture; and, as A B is the true

interſecting line of the ground on which the true meaſures of the objects ſhould be taken, if that were the place of the picture; ſo a b, being the repreſentation of A B, is therefore to be conſidered as the interſecting line of the picture a b z r, on which the proportional meaſures muſt be uſed according to the diminiſhed ſcale. If the picture a b z r were continued down to S I, its interſecting line with the original plane, the repreſentation of the ſpace between S I and a b will fall between S I and a b, and the objects ſituated in that ſpace may be deſcribed either by ſetting off their true meaſures on S I as the interſecting line, or by their proportional meaſures on a b according to the diminiſhed ſcale; for, whether the one or the other of theſe meaſures be uſed on thoſe reſpective lines, ſuch parts of the repreſentation as come within a b z r will, in either caſe, be the ſame; and thus may a picture be formed that ſhall repreſent a very large and extenſive proſpect, the eye being at a moderate diſtance only, by conſidering the picture a b z r as a window in a ſecond or third floor, and the ſpectator as viewing the objects through the ſame; for, in proportion to the height of a b above the ground, and the diſtance E C taken for the eye, the viſible interſecting line A B of the ground, where the deſcription begins, may be removed to any required diſtance, which will have a correſponding effect on the ſpace which is poſſible to be repreſented diſtinctly. As the picture a b z r is a kind of miniature repreſentation of the picture A B Z R, the neareſt objects which are repreſented thereon muſt be leſs than the life, they being ſuppoſed equal to the life at A B: but, if objects were repreſented according to their real dimenſions on the plane a b z r, and thence projected upon the plane A B Z R, they would there appear bigger than the life, which is proper to be done when the diſtance of the eye is neceſſarily very great, as in the caſe of pictures formed on the cieling of a church, &c. where the height is

very great, and the picture designed to exhibit the representations as being nearer than they really are, which it will do, if to this circumstance be added proper and sufficient strength of colour, for that also ought now to be encreased beyond that of real nature, in order to produce the effect here mentioned. In short, if the representations be formed from a scale less, equal to, or greater than the real dimensions of the original objects, they will accordingly appear to the spectator as being farther from, at an equal distance, or nearer than they really are when duly coloured according to those several different circumstances: but, notwithstanding this, it is not necessary, in miniature paintings, to describe the figures in that faintness of colour with which the originals would really appear when so far distant as reduces them apparently to such a size, by means of the angle under which the visual rays do then exhibit them, for a greater distinction of parts and vivacity of colour may be here allowed, taking care only that a due diminution be observed amongst the several objects represented on the picture with respect one to the other; for a picture thus drawn may still be looked upon as such a representation of the objects as would be produced by viewing them through a concave glass, which lessens their apparent magnitudes, but does not take away the distinction of their parts nor their strength of colour in so great a proportion.

OF THE HEIGHT OF THE EYE.

It appears, from what has been shewn, that the height of the eye determines the depth of the original plane, and is always equal thereto, consequently is that which gives bounds to the space which must contain the ichnography of all objects on the original plane that can be represented on the picture; that the image of a line, in a plane parallel to the picture, is of the same length wherever the

eye be placed in the directing plane; therefore the elevating or depreſſing the point of ſight will produce no difference in the apparent heights and breadths of objects, or ſuch of their dimenſions as are parallel to the picture, for they remain of the ſame length, let the height of the eye be what it will, ſo long as its diſtance from the picture remains the ſame; alſo, that the images of any determinate parts of an original line, which inclines to the picture, will have the ſame ratio to each other at all different ſtations of the eye taken in the directing plane, and therefore the altering its height, without changing its diſtance, can have no influence on the apparent decreaſe of the equal parts of the lines which meaſure the depths or diſtances of the objects, by reaſon they have ſtill the ſame proportion one to another, let the height of the eye be what it will, and are effected only as to their being greater or leſs in proportion to the height which is given to the eye.

DEFINITION.

A line or meaſure, taken upon any line parallel to the interſecting line, having the ſame ratio to a given original line as any determinate part of that parallel bears to its original, is called the PROPORTIONAL MEASURE of the original line on that parallel.

LEMMA.

If any determinate part of the indefinite image of an original line inclining to the picture be taken, and a proportional meaſure thereof be found upon a line, drawn parallel to the interſecting line, through that extreme thereof which is neareſt the interſecting line, it will be as the complement of the propoſed image is either equal, greater or leſs than the line producing its vaniſhing point, ſo will the aſſumed

part of that image be accordingly equal, greater, or leſs than its proportional meaſure.

PLATE X. Fig. 11.

Let D K be ſuppoſed the indefinite image of an original line inclining to the picture, and ſuppoſe the line producing its vaniſhing point to be equal to H K; let g m be the determinate part, and g f its proportional meaſure taken on a line parallel to the interſecting line, and paſſing through g, its neareſt extreme to that line; then if m K, the complement of the aſſumed part, be equal to, greater, or leſs than H K, g m will be accordingly equal, greater, or leſs than its proportional meaſure g f.

For ſeeing the triangles H K m, m g f, are ſimilar, it will be

m K : K H :: g m : g f;

therefore, if m K be equal to K H, or whether it be greater or leſſer, the ſame will g m be with reſpect to g f.

PLATE VIII. Fig. 9.

Suppoſe now O N the height of the eye, to be taken ſo great in reſpect of its diſtance O C, that the image q n of any part of an original and inclining line ſhall fall at a greater diſtance from its vaniſhing point, ſuppoſe C, than the length of the line O C which produces its vaniſhing point, then will the image n q be greater than the proportional meaſure of its correſponding original line, when meaſured on a line parallel to the picture paſſing through its neareſt extremity; but, as it muſt ſeem unnatural that the image of a line, inclining to the picture, ſhould appear equal to or greater than the image of a line, of the ſame length, parallel to the picture, and oppoſed directly to the eye, at a diſtance no greater than the neareſt extremity of the ſaid inclining line, ſuch an appearance muſt needs

be deformed and diſagreeable, it ought therefore to be avoided, which it may, by taking ſuch a height for the eye, that the indefinite image of any line in the original plane ſhall not be longer than the line which produces the vaniſhing point of that line, for then, whatever part of the original comes to be deſcribed within thoſe bounds, its image will always be leſs than its proportional meaſure; whence it plainly appears, that the height of the eye has an immediate dependance on its diſtance, which it ought neither to equal nor exceed, but may be leſs at pleaſure, according to the nature of the deſign, and deſire of the artiſt for obtaining more or leſs room for the depths of his objects, ſo as ſhall procure an agreeable proportion between their ichnography and elevations. Nor is the place of the eye always to be conſidered as being the ſame with that of a ſpectator ſtanding upon the original plane, for the eye may be conceived as placed on an eminence, at a conſiderable height above the original plane, ſuch as an hill, the upper part of a building, &c. as when objects, ſituated below, in low grounds, or valleys, &c. are to be deſcribed.

PLATE XI. Fig. 13.

Bring D, in the plane D A B, to coincide with D in the plane N B; make B, in the plane A Z, coincide with B in the plane N B; make the plane S H paſs through the plane D A B in the line F G, and make the plane Q Z paſs through both the planes S H and A Z.

Let now A B M N be the original plane containing the objects to be repreſented, A B the neareſt limits of the ground that is to be deſcribed, then may the ſpectator be placed in d, or any other point of the line E D, provided that the picture can be ſo placed as that the eye, when thus proportionally raiſed, may be on a level with the horizontal line Z R.

OF THE SIZE OF THE PICTURE.

What has been premiſed with reſpect to limiting the greateſt height of the eye relates equally to the ſize of the picture; for, in both, the principal inconvenience, neceſſary to be guarded againſt, is the exceſs of the image of any part of an original line above its proportional meaſure.

PLATE X. Fig. 12.

Let C be the center of the picture, E C the diſtance of the eye; from C, with the radius E C, deſcribe a circle, then if that circle, or any rectilinear figure, D F L P inſcribed therein, be made the bounds of the picture, the image of no line whatever, which is perpendicular to the picture, can, within thoſe limits, be extended farther than the radius, which is the diſtance of the eye from the picture, and therefore, when the principal lines of depth contained in the deſign are perpendicular to the picture, its ſize may be thus determined.

But, when the principal lines of depth incline to the picture, as buildings, having an oblique poſition, &c. then the vaniſhing points of the ichnography of their inclining ſides muſt be taken as centers, and the reſpective diſtances of thoſe vaniſhing points as radii, and circles deſcribed therewith, which, by their mutual interſections, will mark out a ſpace, beyond which no part of the images of thoſe inclining ſides ought to extend.

Thus, ſuppoſe V, v, were the vaniſhing points of the ichnography of the ſides of a building, then taking thoſe points for centers with the diſtance V E or v E, draw the arcs E S e, E T e, interſecting each other in E and e, and the ſpace included thereby will be that in which the repreſentation of the propoſed building ought to be confined.

These rules are to be considered as more immediately relating to the representation of pieces of architecture and animal figures, or such other objects as are of a certain known and determinate form, so as not to suffer their representations to be projected too far distant from the center of the picture; for, as to such other objects as have uncertain, variable and indeterminate forms, as clouds, hills, mountains, and such like, a greater latitude is allowable: for, although what has been before asserted, concerning the necessity of viewing a picture from the exact point of sight, be true, wherever that point be taken, yet, if the distance of that point be too small for the size of the picture, the images of objects near the sides thereof will be extended to great lengths, and take up more space on the picture, than the objects themselves would do if seen directly, and when a picture thus drawn is seen from a different point, the images of those objects will appear deformed or distorted, and be disagreeable to the eye; we should therefore adapt the size of the picture to the distance of the eye, so that none of the representations may appear monstrous or unnatural, wherever the eye be placed to view it; for, notwithstanding that it be from the true point of sight only that a picture can appear exactly as it ought to do, yet, when the distance of the eye is comparatively large with respect to the size of the picture, so that the greatest dimension of the picture may be seen under an angle of ninety degrees or less, any little deviation of the eye from its true place will not have so sensible an effect on the appearance of the picture, as when the distance is smaller, or the picture of a greater extent; and seeing pictures are frequently, if not always, placed in such positions, as that they may be viewed from different points; they ought therefore to be so drawn that, in any of the points fronting them, they may appear as little disagreeable to the eye as is possible, and if nothing contained in them appears remarkably de-

formed, little variations from the ſtrict appearances which they ought to exhibit, will be readily excuſed, and the defect ſupplied by the imagination of the obſerver.

OF THE CONSEQUENCES WHICH ATTEND THE VIEWING OF A PICTURE FROM ANY OTHER THAN THE TRUE POINT OF SIGHT.

It has been before obſerved, that the repreſentations formed on the picture can not appear ſtrictly as they ought to do, if the ſpectator be not in the true point of ſight; and the great perfection of this art conſiſts in forming the picture ſuch, as ſhall excite in the mind of the obſerver the ſame ſenſations as would ariſe therein, by ſeeing the real objects themſelves, which is not indeed to be expected barely from geometrical inveſtigations; recourſe muſt be had to the painter's ſkill in the art of colouring, for rendering the illuſions thus truly perfect: but, nevertheleſs, it is of the utmoſt importance to determine rightly the poſitions which the ſeveral lines and points, terminating the original objects, muſt have on the picture, ſo as to enable them, when duly coloured, to produce the effect wiſhed for by the artiſt; whence the arts of perſpective and colouring ſhould go hand in hand, and afford reciprocal aſſiſtance to each other.

From whatever point it be that a picture is viewed, the repreſentations will appear as the images of objects correſponding to that poſition; whence the ſame repreſentations may be made to allude, by changing the place of the eye, to different original objects, conſequently can not be a true exhibition of the objects there depicted, whoſe real forms, magnitudes and poſitions ought to be invariably preſerved, as ſeen from one certain point, that is to ſay, the true point of ſight; now every different poſition of the eye produces correſponding altera-

tions in the apparent places and other affections of the originals, and diſpoſes the mind to judge of them differently from what they really are in their natural ſtate; for different poſitions of the eye, taken in the line which is the parallel of the eye, belonging to the plane containing the original objects, will produce alterations of one ſort, while others, taken in the eye's director, occaſion changes of another kind; and the placing the eye nearer to or farther from the picture than it ought to be, cauſes variations different from either, as we ſhall now endeavour to make appear, by a few general obſervations derived from what has been before delivered.

And firſt, let us ſuppoſe the eye of the ſpectator to be placed ſomewhere in the parallel of the eye out of the true point of ſight, then will the center of the vaniſhing line appear to be as far diſtant from the true center as is the ſuppoſed place of the eye from the true place, and on the ſame ſide thereof; but the vaniſhing line of the plane which belongs to that parallel of the eye, as alſo the height of the eye, will remain without alteration, and thoſe dimenſions of objects, which are parallel to the picture, will preſerve their true magnitudes and poſitions: if the original plane be horizontal, it will ſtill appear ſo; the elevations of objects, ſituated thereon, will continue to appear perpendicular to it, and the ſame judgment will be paſſed on all lines in the ichnography which are parallel to the picture, as would be done from the true point of ſight; but the angles, formed by lines which incline, the planes raiſed upon them, and the apparent meaſures of their ſeveral parts, will be different from what they really ſhould be. For the true center of the vaniſhing line is the vaniſhing point of the images of all lines in the ichnography which are perpendicular to the interſecting line, and when the apparent place thereof is changed, that true center then becomes the vaniſhing point of ſuch original lines as incline to the line of ſtation,

or line which determines the poſition of the ichnography, ſo as to form therewith an angle, whoſe tangent is the diſtance between the real and apparent centers, the diſtance of the eye being made radius, and a correſpondent change will enſue in the appearance of all other original lines, whoſe images tend to any other vaniſhing points, and conſequently in the apparent inclinations of any elevated planes, whoſe vaniſhing lines paſs through thoſe points.

PLATE IX. Fig. 10.

Suppoſe now the eye to be placed in e, and draw eV perpendicular to the vaniſhing line H L, then will V be the apparent center; for, in whatever point the eye be placed, a line drawn from thence perpendicular to the picture, the point wherein that line cuts it, will be judged to be its center; and x C, the whole image of an original line, perpendicular to the interſecting line, will appear as inclining to the line of ſtation, which is parallel to E C, under the angle V e C; now, if e V or E C be aſſumed the radius, then is V C the tangent of the ſaid angle V e C. And if e V or E C were the vaniſhing line of a plane paſſing through x C perpendicular to the picture and original plane, it will appear, from the ſtation e, as a plane perpendicular indeed to the original plane, but it will ſeem as inclining to the picture in the angle e C V: hence it is that a plane, which would appear perpendicular to the picture when ſeen by an eye at E, will, as the eye is removed in the line E X, ſeem to incline more and more towards the picture, and always the contrary way to that in which the eye is tranſlated from one place to the other; for inſtance, the plane ſeems to incline towards Z, as the eye moves from E towards X, and when the eye moves from E towards Z, then it ſeems to incline in like manner towards X.

If any two vaniſhing points, H and h, be taken, and there be drawn from the true point of ſight the lines E H, E h, they will ſubtend the true viſual angle under which thoſe two points will appear; alſo drawing the lines e H, e h, they will ſubtend the angle under which the ſame points will appear from the place or ſtation e, and conſequently all original lines tending to thoſe points will appear at e, under the angle H e h, which will be equal, greater or leſs than the true angle, according where the points H and h happen to meet the vaniſhing line H L, with reſpect to the real and apparent centers C and V.

For if, upon the line H h, ſuch a ſegment of a circle be deſcribed on the vaniſhing plane, which, paſſing through E, contains the true viſual angle H E h, (33. 3.) it is plain that, in whatever point of its circumference the eye be placed, the angle ſubtended by lines drawn from thence to the points H and h, will appear the ſame; therefore if the eye be at q, the point where it cuts the parallel of the eye, the apparent angle H q h will be equal to the true angle; but, if the eye be in or out of the circle bounded thereby, the apparent angle will be accordingly greater or leſs than the true angle H E h.

Now, as the line x C appears at the point e to incline to the picture, the line e C becomes that which apparently produces its vaniſhing point; and therefore, if x C be any how divided, ſuppoſe in the points n, c, &c. the apparent meaſures of the parts x n, n c, &c. will be increaſed or diminiſhed in the ſame proportion which the apparent line e C has to the true line E C, which really produces the vaniſhing point of that line; ſo that, inſtead of repreſenting their true meaſures x I, I J, from the place e, the mind will judge them as being equal to x w, w l. And ſeeing that the apparent line, producing the vaniſhing point, may be equal, greater or leſs than the true one, according to the poſition which any vaniſhing point in the

line H L has to the apparent center V, excepting only the point C, for the apparent line belonging thereto can never be leſs than E C, it follows, that the images of the parts of any inclining original line may accordingly repreſent parts equal, greater or leſs than their true originals; but, notwithſtanding this, the perpendicular diſtance between the picture and the points n, c, &c. will appear true, ſo long as the eye continues in its parallel X Z. The ſame holds good alſo with reſpect to any elevated plane, whoſe vaniſhing line is E C, or a line parallel thereto; for, although its center is not varied while the eye moves in the line X Z, yet the line, which apparently determines its vaniſhing point, becomes equal to e C, and as that is greater than the true one, a correſponding effect will be thereby wrought on the apparent lines belonging to all other vaniſhing points in that line, as alſo on the apparent angles and diſtances from the interſecting line of the originals of any points in that elevated plane, though not on their perpendicular diſtances from the picture.

The eye being placed in any point of the director of any original plane, thoſe dimenſions of objects, which are parallel to the picture, appear unvaried both with reſpect to their magnitude and diſtance as above-mentioned; but the apparent place of the horizon will be changed, and ſeem as being above or below the true horizontal line, according as the eye is higher or lower than the true point of ſight; the original plane will accordingly appear as raiſed above or ſunk below the horizon, and the perpendicular ſupports of all original points appear as oblique ſupports; alſo the apparent line, producing the vaniſhing point, will be increaſed, by altering the apparent place of the center of the picture, the originals of the parts of all inclining original lines will be judged proportionably greater than they really are, and the objects will thereby ſeem to occupy more ſpace in depth, on this inclining plane, than they actually do, though their perpen-

dicular diſtances from the picture are not affected. Now, ſeeing the eye is ſuppoſed always in the ſame director, the vertical line of the original plane will remain unaltered, and conſequently all elevated planes will appear inclining to the picture under the ſame angles as they would do from the true point of ſight, but will not appear perpendicular to the original plane, but as the planes of the oblique ſeats of lines on that plane, and thoſe lines in the planes of elevation which ſhould appear parallel to the horizon, ſuch as the ranges of windows, cornices, or other members in buildings of architecture, by the apparent change of their centers, will appear elevated or depreſſed above or below the horizon, according as the height of the eye is greater or leſs than that of the true point of ſight. Alſo the apparent angles, formed by inclining lines in thoſe planes, and the apparent magnitudes of their parts, will be in like manner affected from the apparent changes in the centers, made by varying the lines which apparently produce the vaniſhing points thereof. But ſo long as the eye continues in the ſame director, the perpendicular diſtances betwen the picture and all the points in thoſe planes continue to appear as they ſhould do, by reaſon the director is, in this caſe, the parallel of the eye with reſpect to the ſaid elevated planes.

But, if the eye be placed any where in the line producing the vaniſhing point of any original line, farther from or nearer to the picture than the true point of ſight, the center of the picture, horizontal and vertical lines will retain their true appearance, and all planes of elevation will continue to appear perpendicular to the original plane; but the diſtance of the eye, being thus varied, will have an effect upon the apparent magnitudes of the parts of all inclining original lines, and conſequently on the apparent diſtances of thoſe parts from the picture, and likewiſe on the apparent angles ſubtended by lines joining the vaniſhing points of lines in that plane,

for they will appear greater or leſs than they truly ought, according as the diſtance of the eye is leſſened or increaſed; alſo the apparent inclinations of all elevated planes, and lines therein, with reſpect to each other and the picture, will be affected, except only the right angle contained by the parallels and perpendiculars to the interſecting lines of the ſeveral planes, for they will all continue to appear perpendicular to each other, wherever the eye be placed in the line here mentioned, becauſe the center of the picture continues the ſame, the centers of all vaniſhing lines in the picture will remain unvaried; and the magnitudes of the parts of thoſe parallels, and of all other lines which meaſure the dimenſions of objects parallel to the picture, will be judged the ſame.

PLATE VIII. Fig. 9.

Thus, ſuppoſe S H to be the picture, C its center, O C its diſtance, O being now taken for the true point of ſight, if that diſtance be increaſed as to E, the apparent line above-mentioned will then be E C, and the apparent angle L E H, ſubtended by the line joining the vaniſhing points L and H, will be leſs than the true angle L O H, and would be more ſo, were the eye in ſome other point of that line between O and C, and the reſt, it is plain, is as above declared.

Now, let Y E be the vertical plane, C the center of the picture, O C its true diſtance, D F the line of ſtation; let A B be an original line in that plane perpendicular to the original plane, and parallel to the picture, whoſe image, ſeen from O, the true point of ſight, is k y or q y. Let now the eye be ſuppoſed in E, then will K U, which is the apparent original of the image k y, although it appears farther diſtant from the picture than its true original A B, be judged,

by the mind of the ſpectator, to be of the ſame ſize as A B; that is, A B and K U will be equal to each other.

For, in the ſimilar triangles E O k, k A K,

$$E\,k : k\,K :: O\,k : k\,A,$$

by compoſition $E\,k + k\,K = E\,K : E\,k :: O\,k + k\,A = O\,A : O\,k.$

But, in the ſimilar triangles E k y, E K U,

$$E\,K : E\,k :: K\,U : k\,y,$$

and, in the ſimilar triangles O k y, O A B,

$$O\,A : O\,k :: A\,B : k\,y;$$

therefore, by a parity of reaſoning,

$$K\,U : k\,y :: A\,B : k\,y,$$

conſequently $K\,U = A\,B.$

If we ſuppoſe E the true point of ſight, and O the point aſſumed for the place of the eye, the ſame reaſoning will ſhew, that the diſtance of the eye being leſſened, the original of k y will be judged nearer the picture than it really is, but ſtill of the ſame magnitude.

Now, ſeeing it is as A B to k y, ſo are all other lines in a plane parallel to the picture paſſing through A B to their images ſeen from the ſame point O, and likewiſe as K U is to k y, ſo are all other lines in a plane parallel to the picture, paſſing through K U to their images ſeen from E; therefore, ſeeing A B and K U are equal, the originals of all lines in the picture, which meaſure the dimenſions of objects parallel to it, will be judged of the ſame length, let the eye be placed where it will in the line O C or E C.

What has been advanced relating to the ſeveral poſitions of the eye as being placed in its parallel, director, or as laſt-mentioned, is eaſily applied to any other poſition of the eye whatever; for, ſuppoſing it placed any where in the directing plane, the conſequences will be derived from what has been ſaid of the parallel and director,

and, if placed any where out of the directing plane, by taking in the above-mentioned conſiderations, the effects will readily appear.

Upon the whole, we may obſerve, that the ill effects produced on the picture, by its not being viewed from the true point of ſight, are not ſo conſiderable by its being ſeen from a point that is too near or too far, ſo long as the eye remains on a level with the horizontal line, as are thoſe which ſpring from a poſition of the eye that is higher or lower than it ought to be; and that a reaſonable licence may be allowed for deviating from the true point of ſight in viewing ſuch pictures as are on a plane ſurface, without incurring the fault of any other inconſiſtencies or miſrepreſentations, but what may, in ſome ſort, be corrected by the imagination of the ſpectator; but, as to thoſe pictures which are formed on other kinds of ſurfaces, as domes, or arched roofs, or other irregular figures, it is there abſolutely neceſſary to pay the ſtricteſt regard to the true point of ſight; for any the leaſt variation from it, occaſions the moſt groſs appearances, and makes the figures ſeem as being diſjointed, broken or diſtorted, and the unity of the repreſentation is thereby deſtroyed.

Having far exceeded the bounds firſt preſcribed to this little Tract, we here conclude, reſerving the doctrine of ſhadows, the method of applying perſpective to ſcenographical repreſentations, as now practiſed in painting the ſcenes of theatres, the manner of drawing anamorphoſes or deformed appearances, &c. for a future diſquiſition.

FINIS.

Plate I.

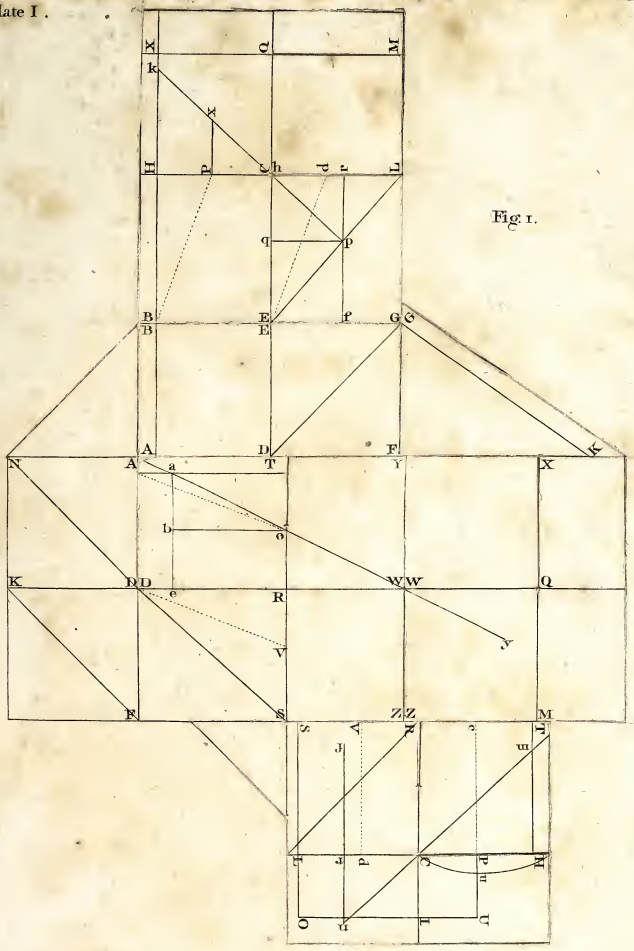
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Fig. 1.

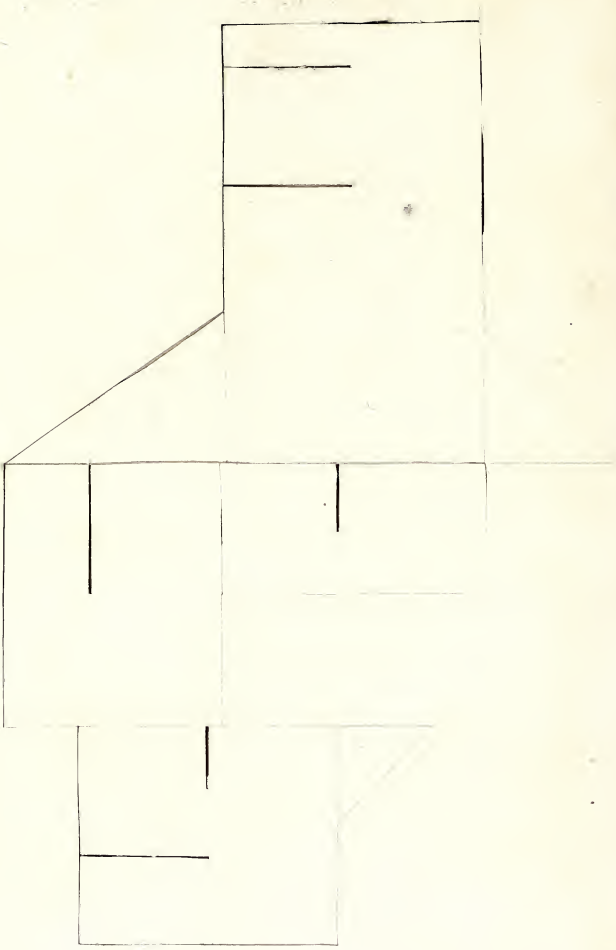


Plate II.

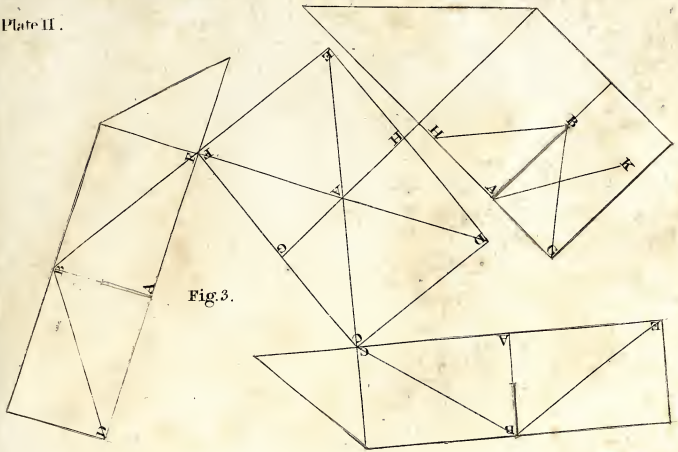


Fig. 3.

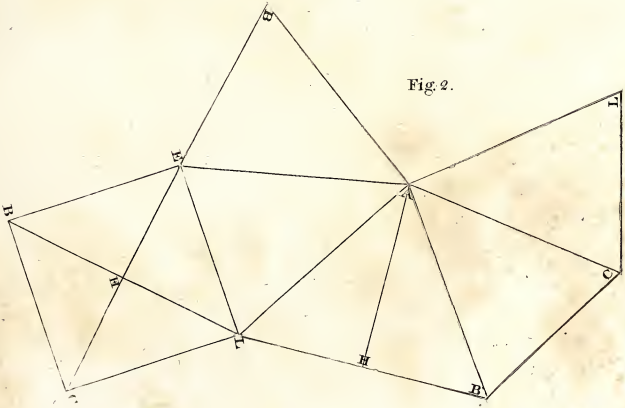


Fig. 2.

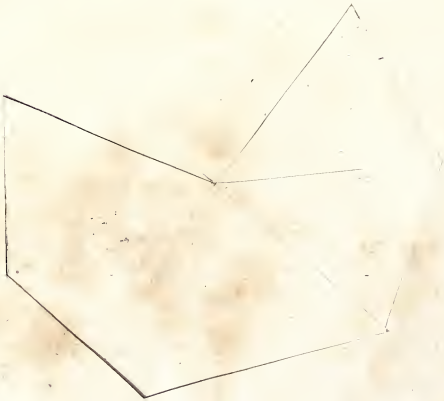


Plate III.

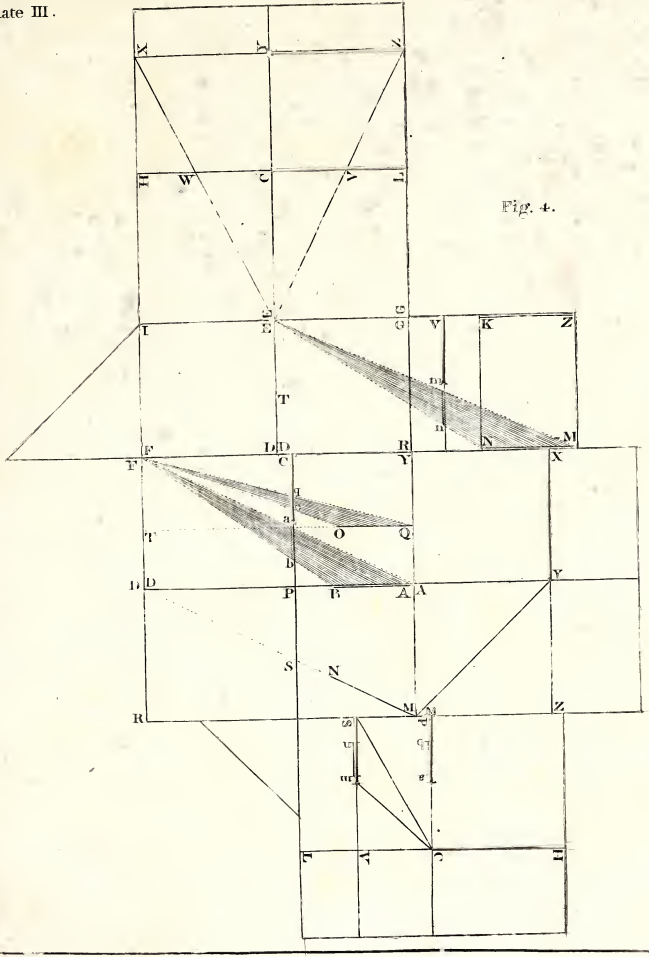


Fig. 4.

Fig. 5.

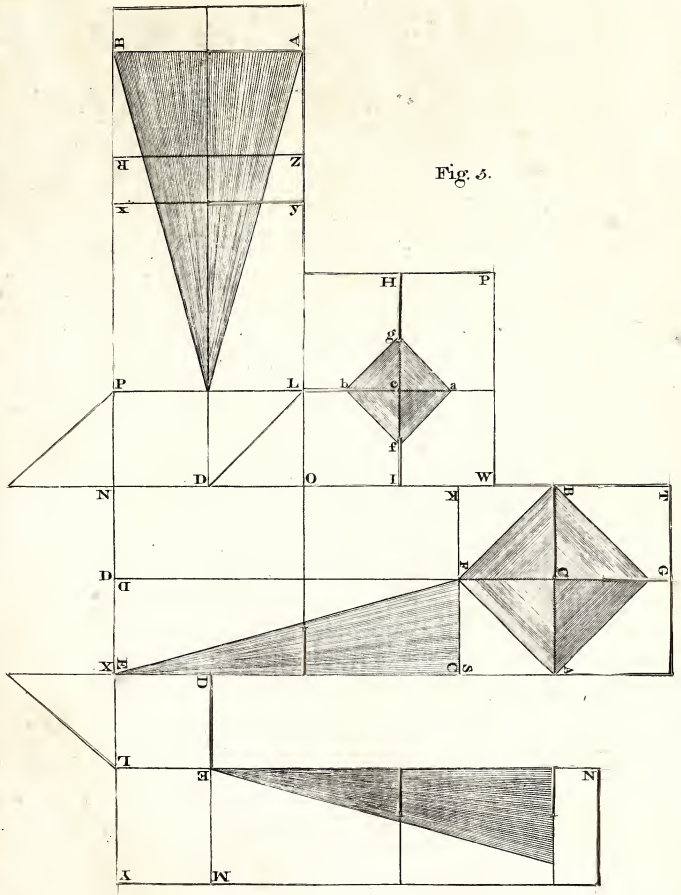
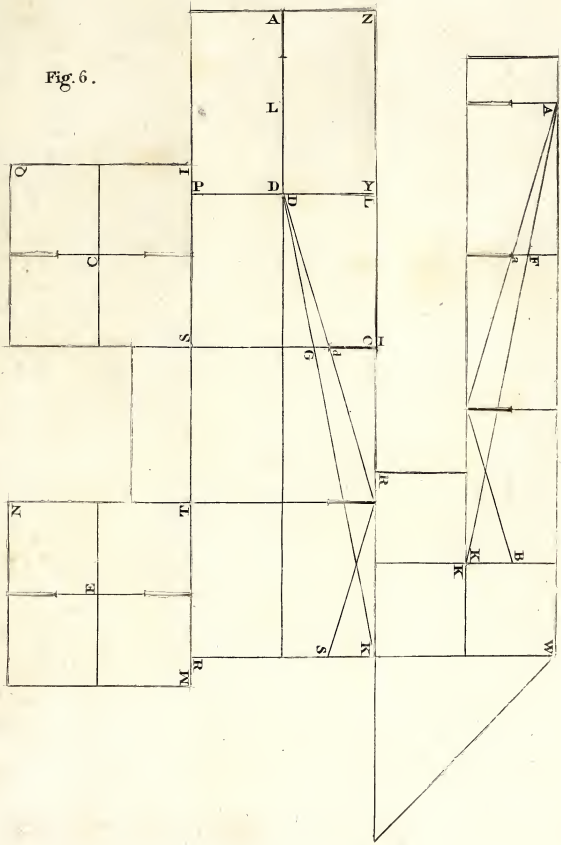
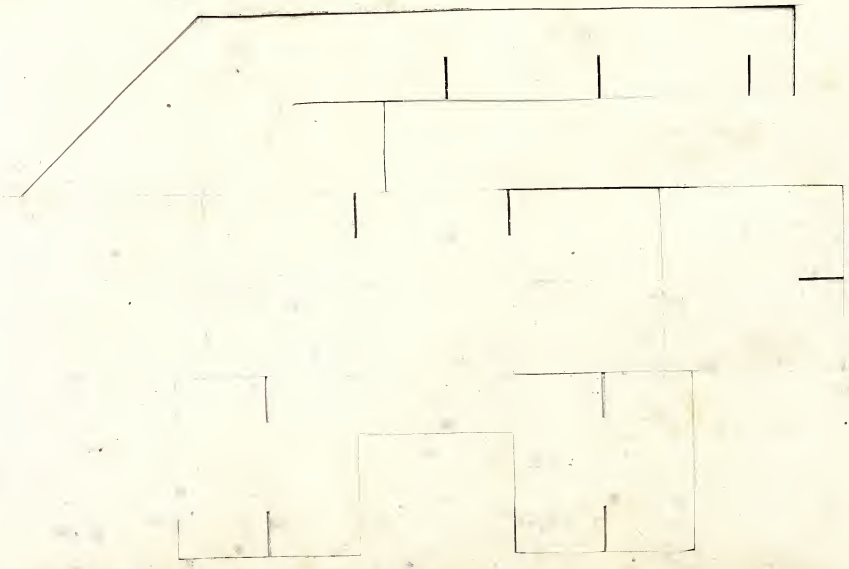
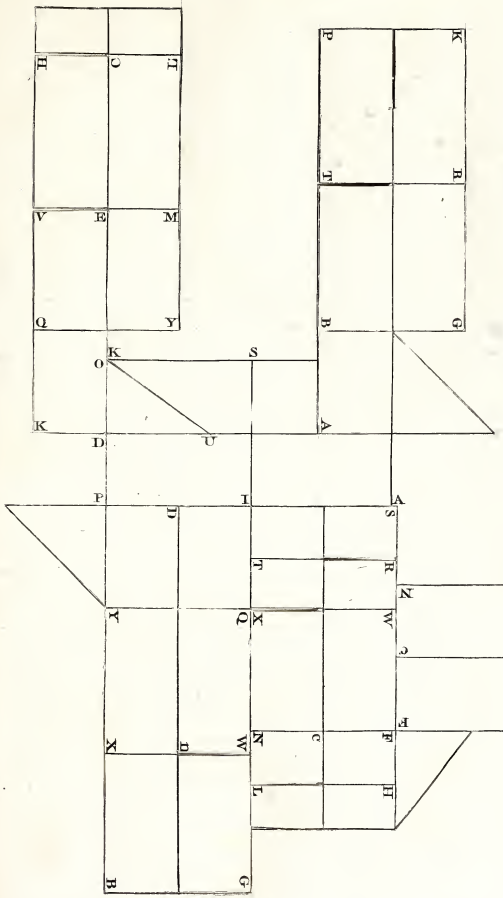


Fig. 6.







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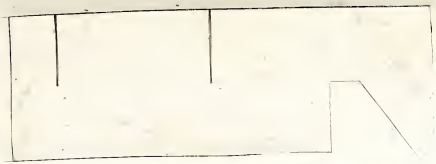
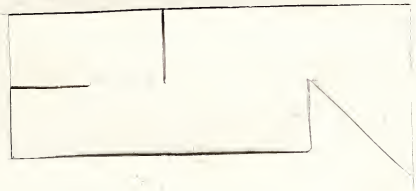




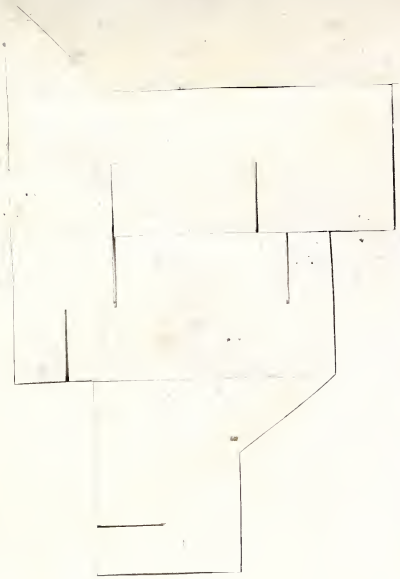
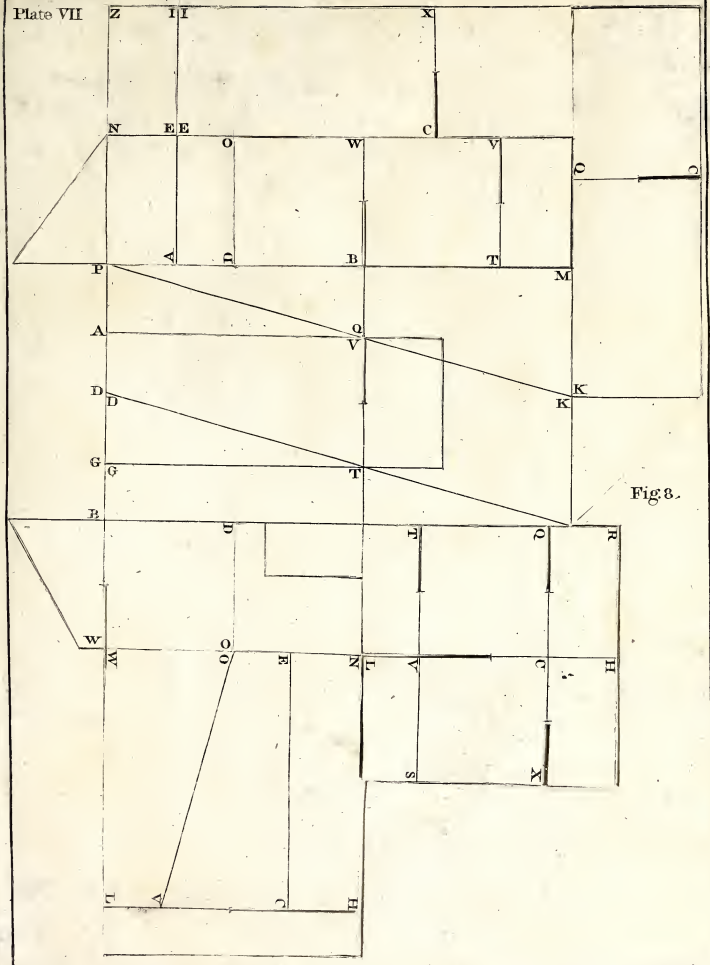


Plate VII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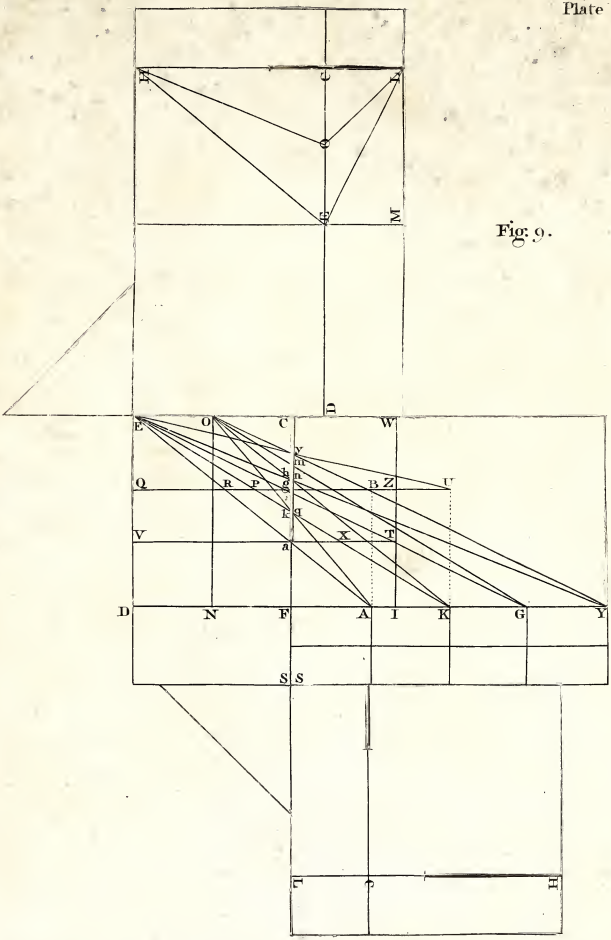
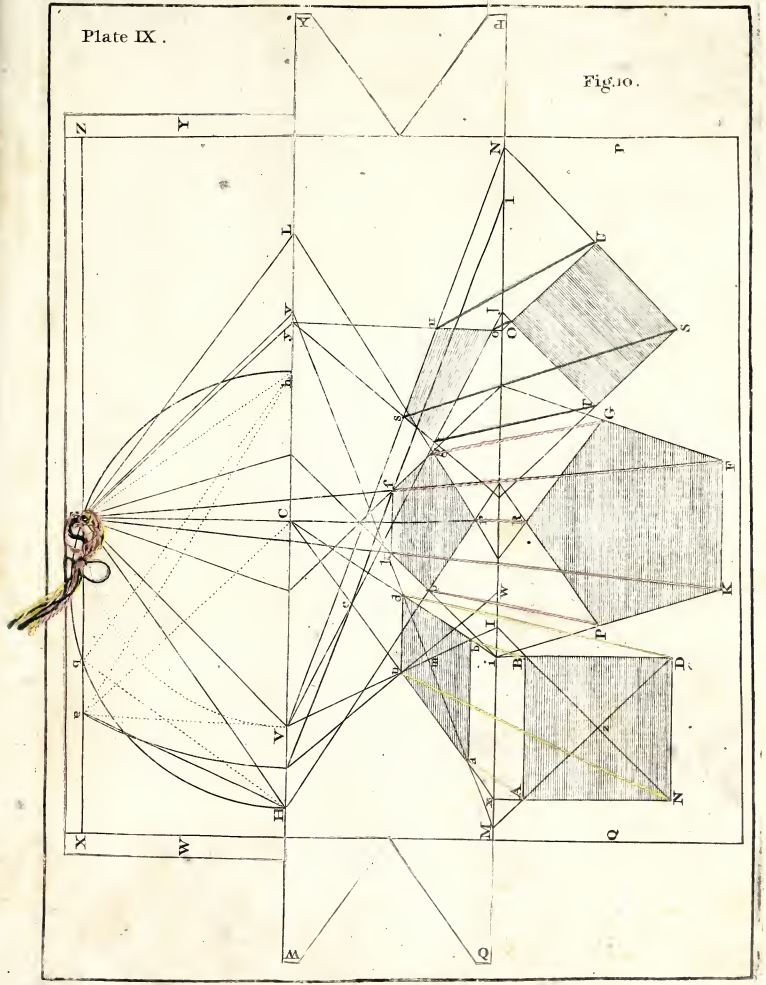
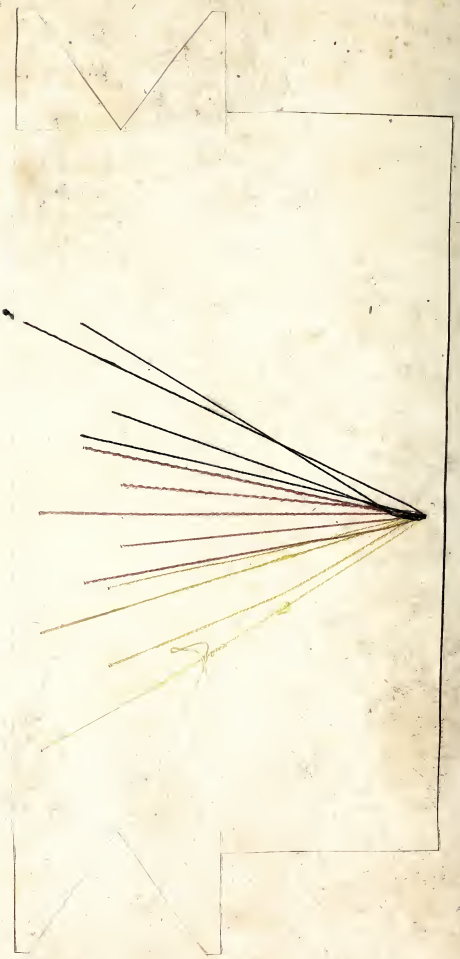


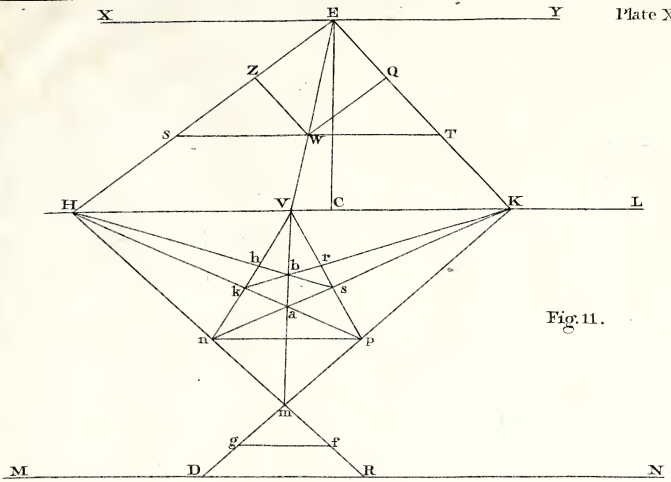
Fig: 9.

Plate IX.

Fig. 10.







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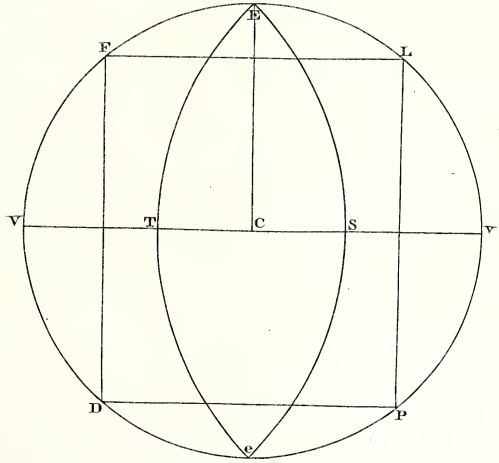
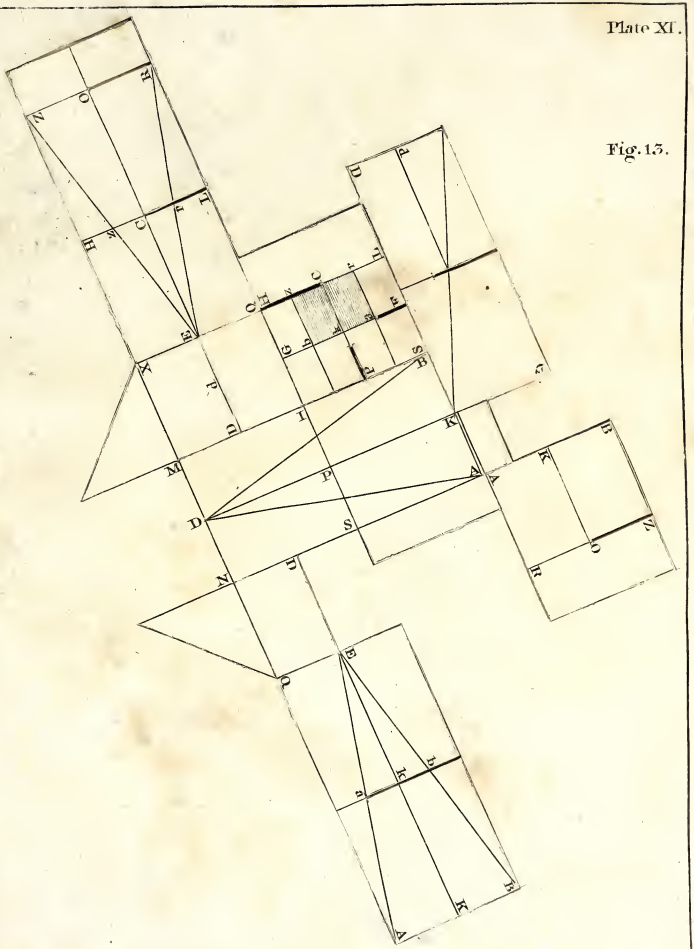


Fig. 12.

Fig. 13.



[6], [xii], 117; [3] pp.

[*]^{6}, a-c^{2}, B-Z^{2}, Aa-FP^{2}, 6g^{4}

With 11 Full page cut-out plates of geometrical figures

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